RYERSON UNIVERSITY
Department of Electrical and Computer Engineering

ELE 635
Communication Systems

Phase Locked Loop and Carrier Recovery

Winter 2015

In this week's lectures we will discuss **two topics** that are fundamental in the design of many communication systems.

These topics are:

- Phase Locked Loop (PLL).
- Carrier Recovery/Acquisition.

Phase Locked Loop

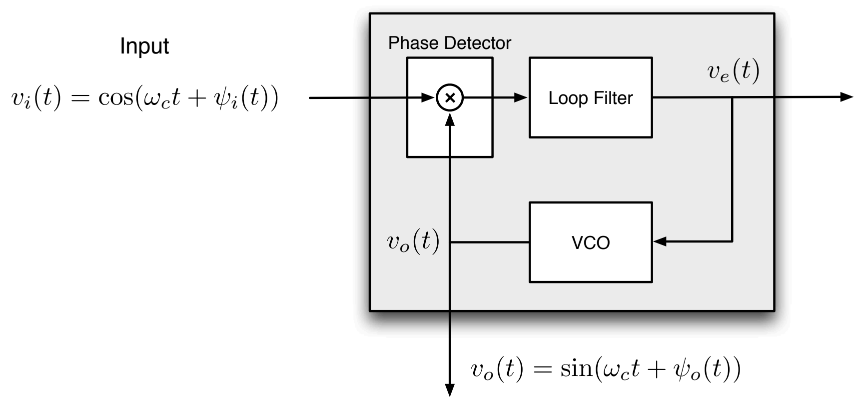
PLL

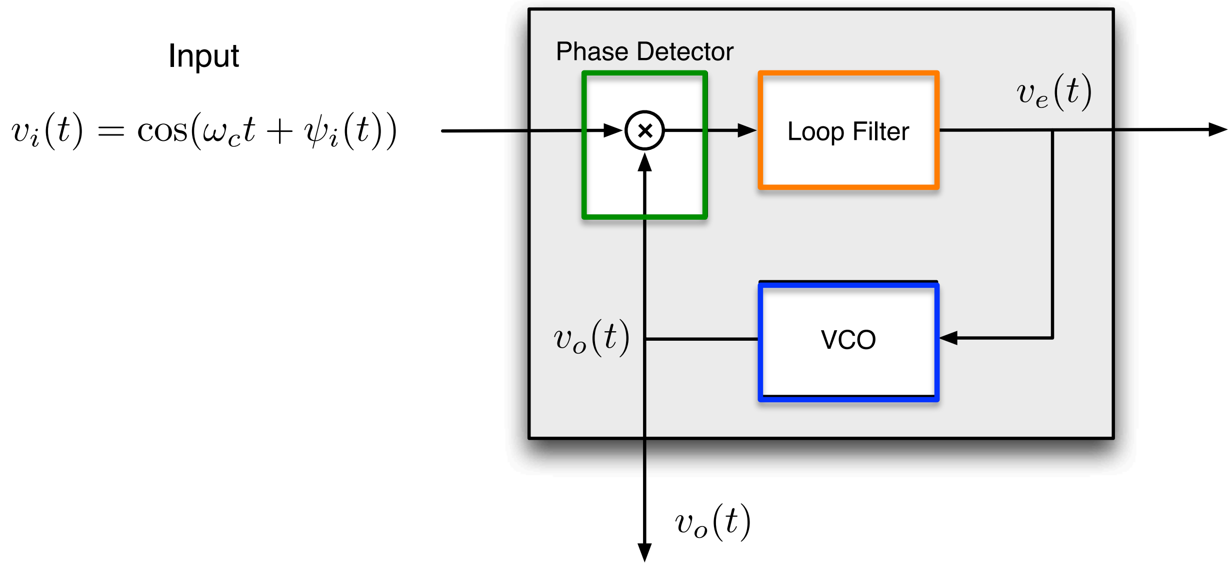
What is a Phase Locked Loop (PLL)?

The **PLL** is a control system that is used to track the (potentially) time-varying phase and the instantaneous frequency of the carrier component of a signal.

It is an extremely useful system that allow synchronous (coherent) demodulation of modulated signals. It is also a fundamental component in the demodulation of angle modulated (PM and FM) signals.

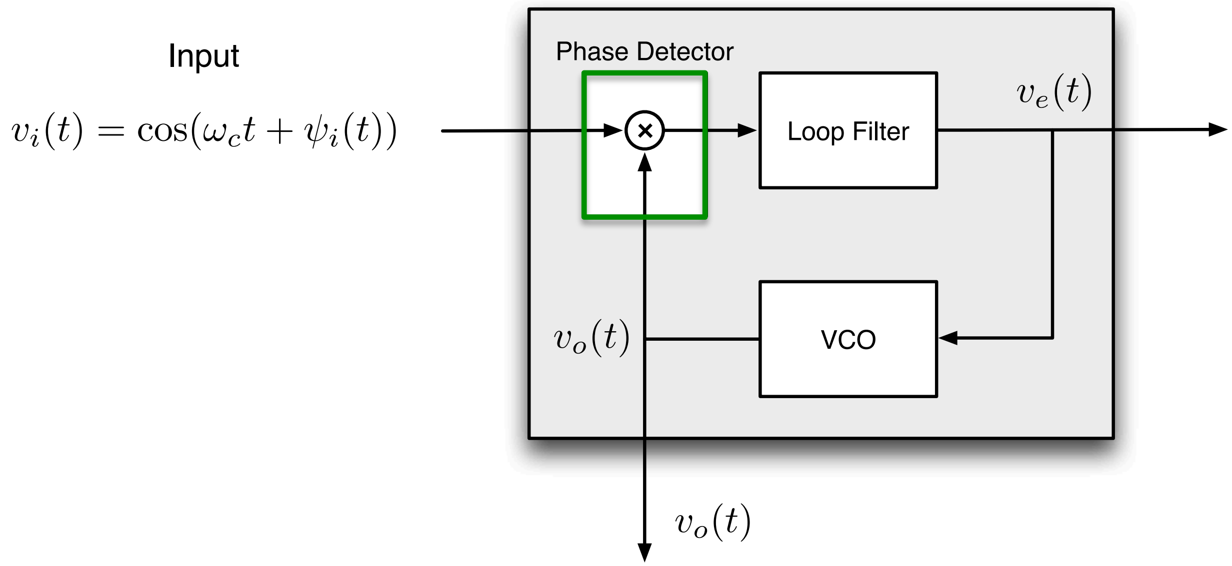
PLLs are also used in the frequency synthesis and detection of FSK-formatted digital signals.





PLL has three functional units:

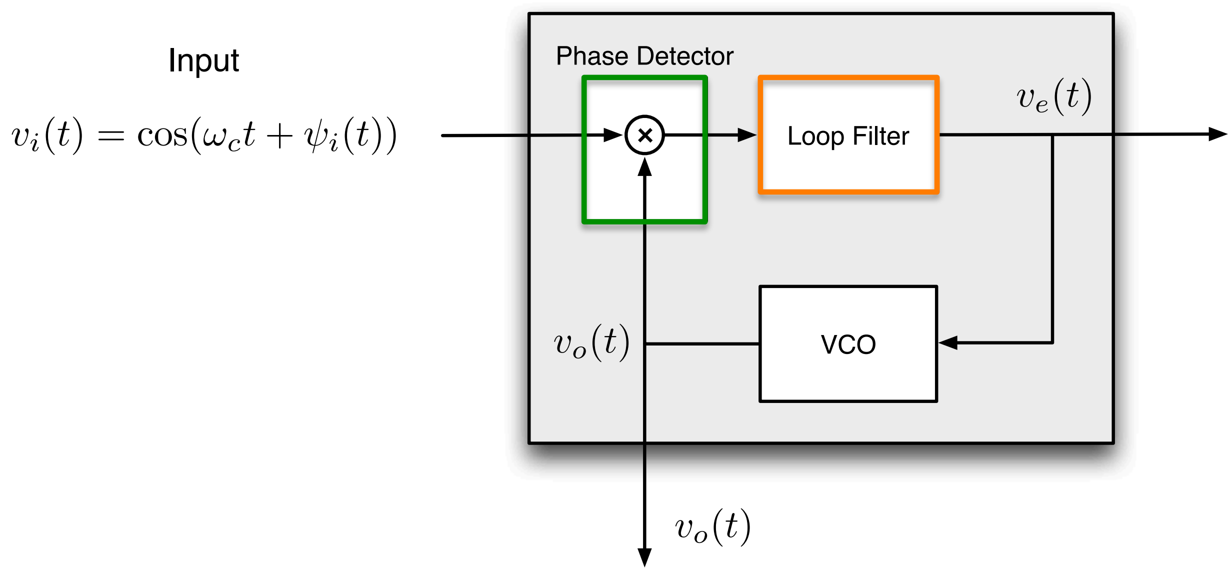
- Phase Detector
- Loop Filter
- Voltage controlled oscillator **VCO**



PLL has three functional units:

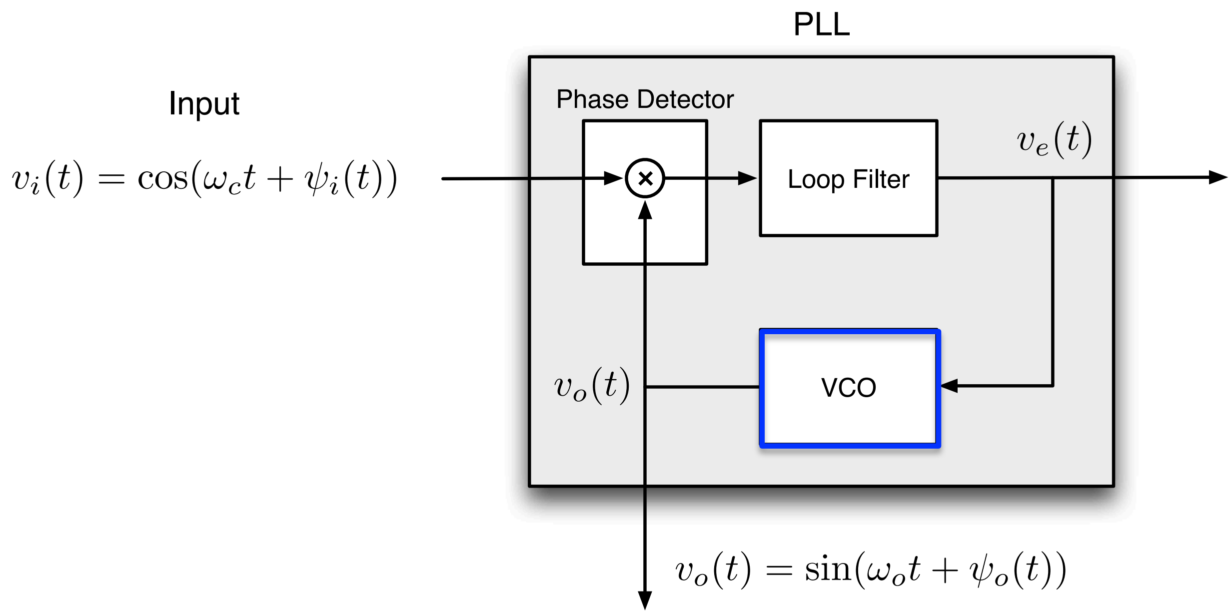
- **Phase Detector**

  PD acts as a multiplier, to be more precise the PD includes a multiplier, a filter and an amplitude scaling unit. For our initial discussion it is sufficient to consider the PD unit as a multiplier only.



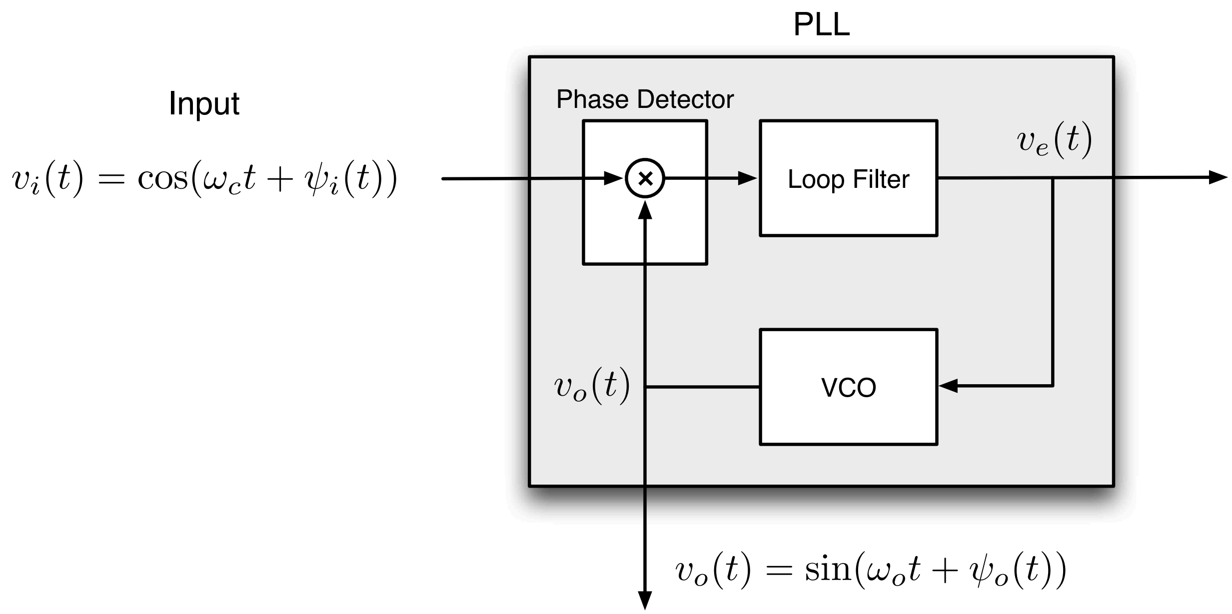
PLL has three functional units:

- Phase Detector
- **Loop Filter** The loop filter is a narrowband lowpass filter.

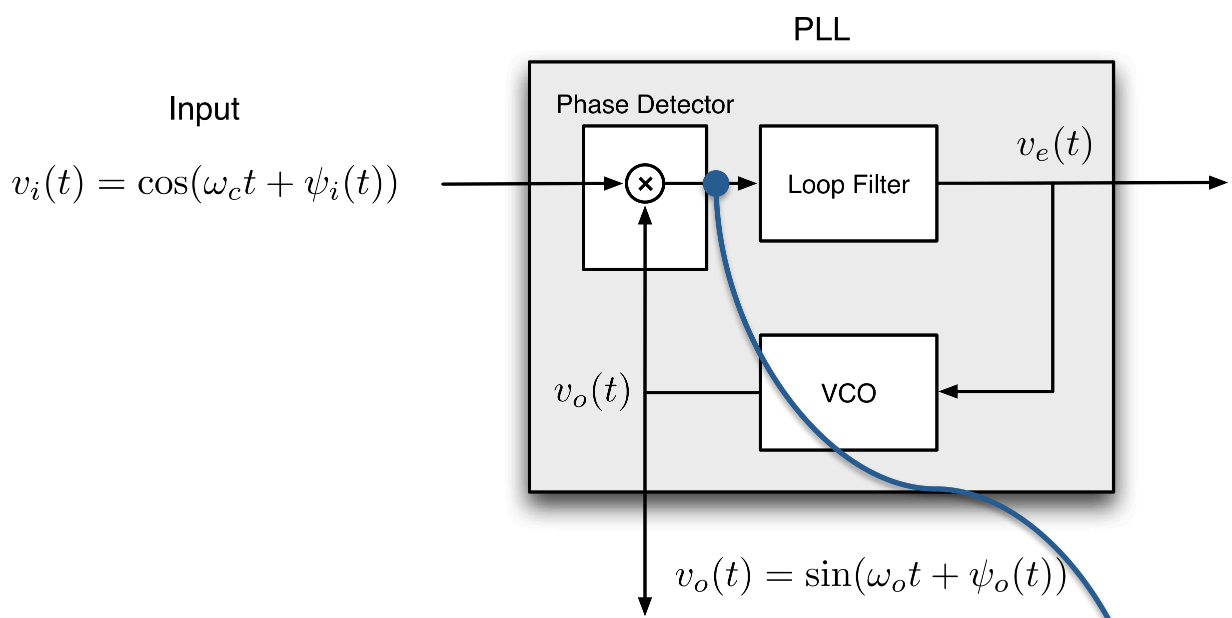


PLL has three functional units:

- Phase Detector
- **Loop Filter**
- Voltage controlled oscillator (**VCO**) an oscillator whose output can be controlled by the voltage level at its input $v_e(t)$



- PLL functions as a **closed loop feedback system**.
- Objectives:
  - **Lock** the VCO frequency to that of the incoming signal
  - **Track** changes in the **instantaneous frequency** of $v_i(t)$.

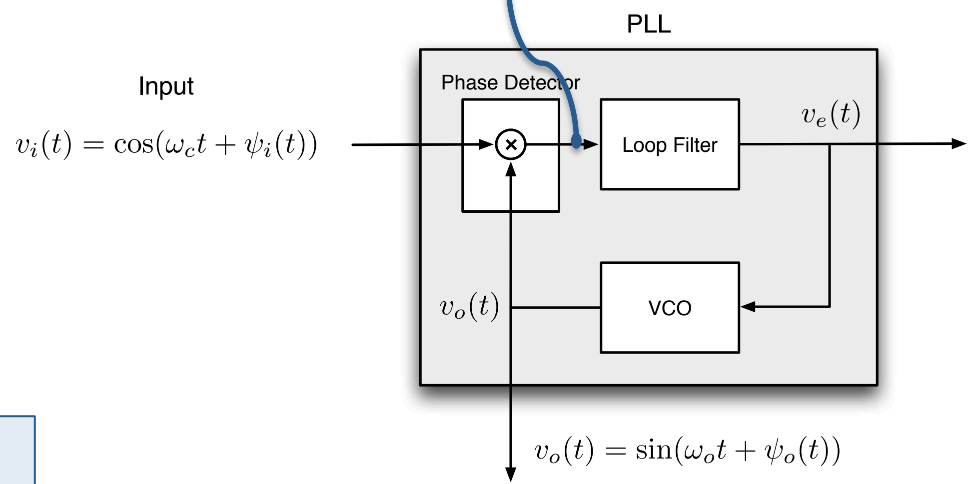


Phase detector output:

$$v_i(t)v_o(t) \sim \cos(\omega_c t + \psi_i)\sin(\omega_o t + \psi_o)$$

$$\sim \sin\big((\omega_c - \omega_o)t + (\psi_i - \psi_o)\big) + \sin\big((\omega_c + \omega_o)t + (\psi_i + \psi_o)\big)$$

$$v_i(t)v_o(t) = 0$$

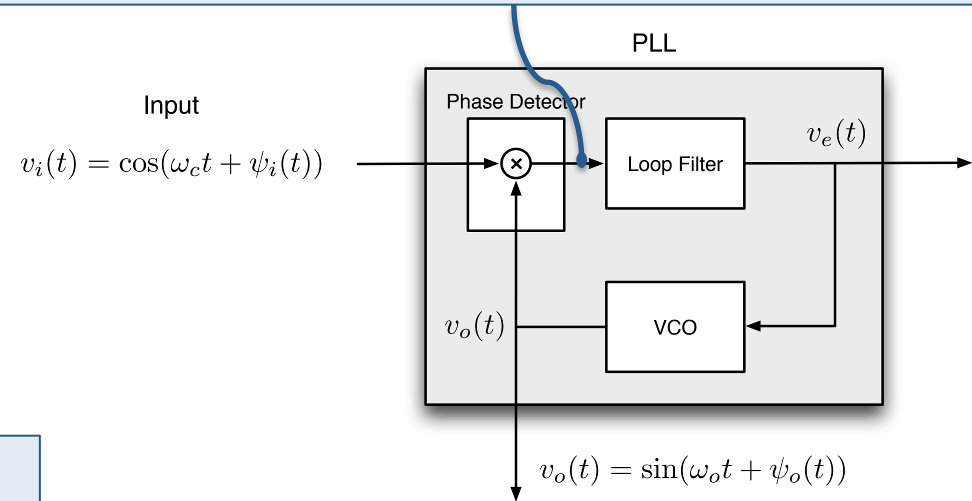


$$v_e(t) = 0$$

If $v_i(t) = 0$

- $v_i(t)v_o(t) = 0$
- Loop filter output: $v_e(t) = 0$
- VCO continues to operate at its free running frequency f_0

$$v_i(t)v_o(t) \sim \sin\big((\omega_c - \omega_o)t + (\psi_i - \psi_o)\big) + \sin\big((\omega_c + \omega_o)t + (\psi_i + \psi_o)\big)$$



If $v_i(t) \neq 0$

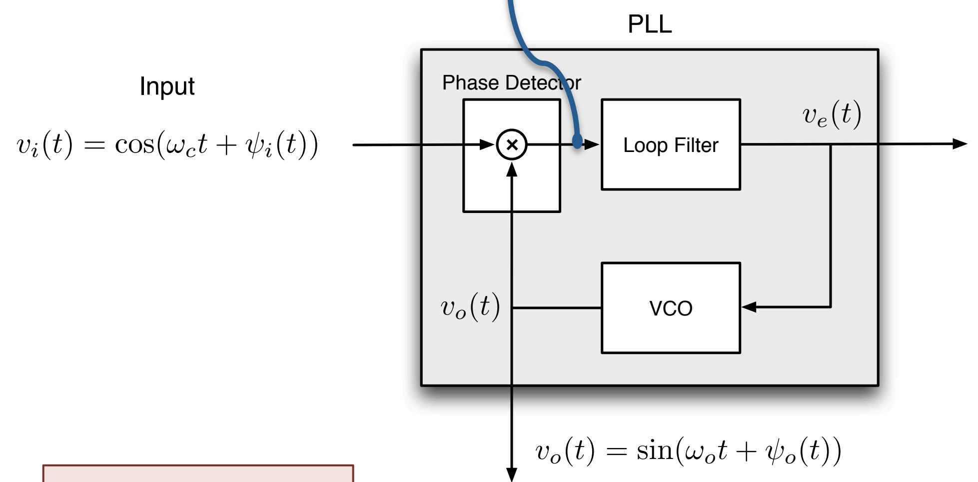
- $v_i(t)v_o(t) \neq 0$
- PD generates the signal $v_i(t)v_o(t)$ with frequency components at $f_c \pm f_0$

Which of the two frequency components at

$f_c \pm f_0$

will make it through the Loop Filter?

$$v_i(t)v_o(t) \sim \sin\big((\omega_c - \omega_o)t + (\psi_i - \psi_o)\big) + \sin\big((\omega_c + \omega_o)t + (\psi_i + \psi_o)\big)$$

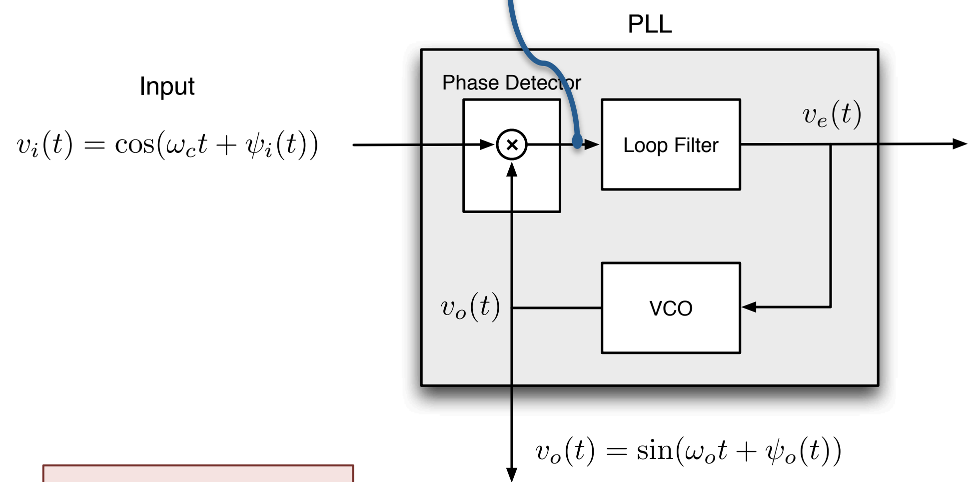


$v_e(t) = 0$

If $v_i(t) \neq 0$ and $|f_0 - f_c| > \gamma$

- Both frequency components in $v_i(t)v_o(t)$ will be eliminated by the loop filter.
- Loop filter output: $v_e(t) = 0$
- VCO continues to operate at its free running frequency.

$$v_i(t)v_o(t) \sim \sin\big((\omega_c - \omega_o)t + (\psi_i - \psi_o)\big) + \sin\big((\omega_c + \omega_o)t + (\psi_i + \psi_o)\big)$$

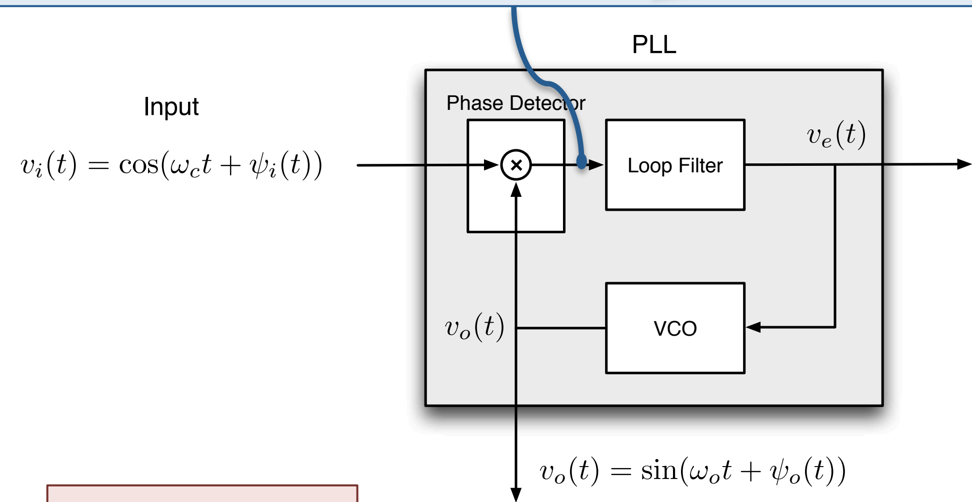


$v_e(t) \neq 0$

If $v_i(t) \neq 0$ and $|f_0 - f_c| < \gamma$

- Frequency component at $f_c + f_0$ will be eliminated by the loop filter.
- Frequency component at $f_c - f_0$ will be within the passband of the loop filter.
- Loop filter output: $v_e(t) \neq 0$
- VCO will change its output … How?

$$v_i(t)v_o(t) \sim \sin\big((\omega_c - \omega_o)t + (\psi_i - \psi_o)\big) + \sin\big((\omega_c + \omega_o)t + (\psi_i + \psi_o)\big)$$



$v_e(t) \neq 0$

If $v_i(t) \neq 0$ and $|f_0 - f_c| < \gamma$

- $v_e(t) \neq 0$ ⟹ VCO frequency changes to reduce the error signal.
- $|f_0 - f_c| < \gamma$ ⟹ VCO will lock to the instantaneous frequency of $v_i(t)$
  ⟹ PLL will track the instantaneous frequency of $v_i(t)$

Given that

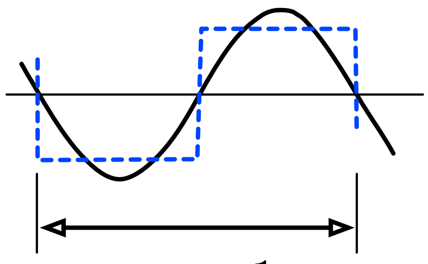
$$v_i(t) = \cos\big(2\pi f_c t + \psi_i(t)\big)$$

Initial PLL set-up

- Set VCO frequency to equal to the carrier frequency → $f_0 = f_c$
- VCO output has 90° phase shift with respect to the unmodulated carrier:
  - unmodulated carrier → $\cos 2\pi f_c t$
  - free-running VCO output → $\sin 2\pi f_0 t = \sin 2\pi f_c t$

  such that PD output equals 0.
- The VCO waveform used in most PLLs is a square-wave.

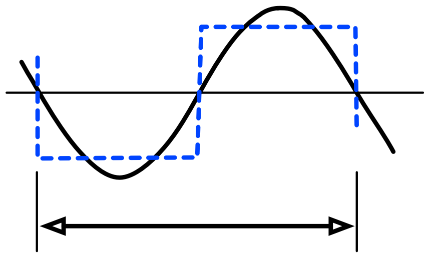
Why use a Square Wave?



$$T_c = \frac{1}{f_c}$$

- A square wave is easier to generate

Why use a Square Wave?



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- Given the more accurate phase detector model:

PD = Multiply + Filter + Amplitude Scale

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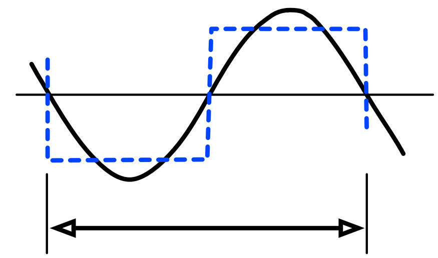

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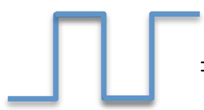
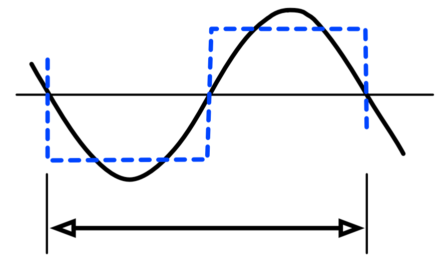
and the representation of a square wave as:



$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\big(2\pi(2n-1)f_0 t\big)$$

Why use a Square Wave?



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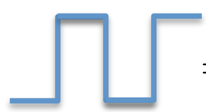
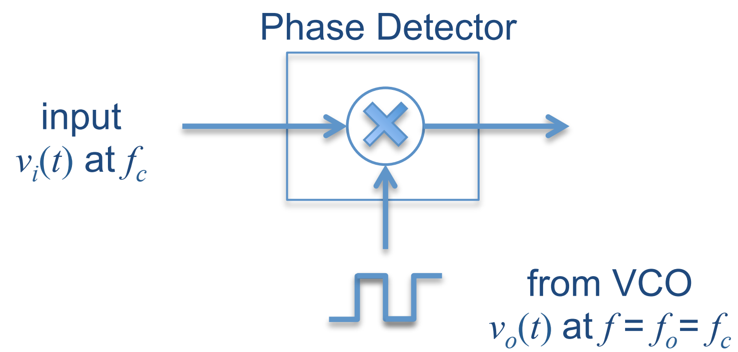


$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\big(2\pi(2n-1)f_0 t\big)$$

after multiplication and filtering, the PD output will only include the fundamental term → $\sin 2\pi f_0 t$

Why the 90º phase shift ?



Case 1

$$\left.\begin{aligned} v_i(t) &= \cos 2\pi f_c t \\ v_o(t) &= \cos 2\pi f_o t = \cos 2\pi f_c t \end{aligned}\right\} \psi_e = \psi_0(t) - \psi_i(t) = 0$$

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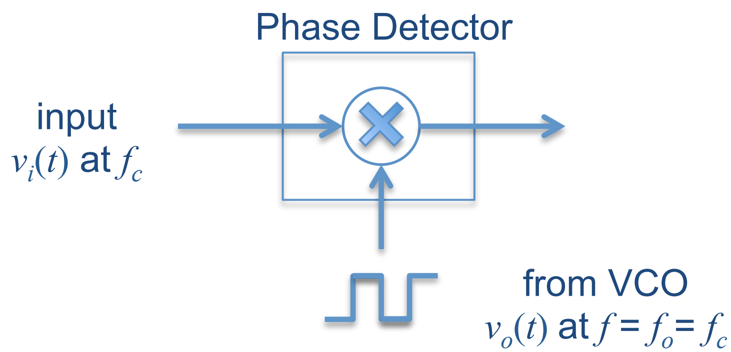

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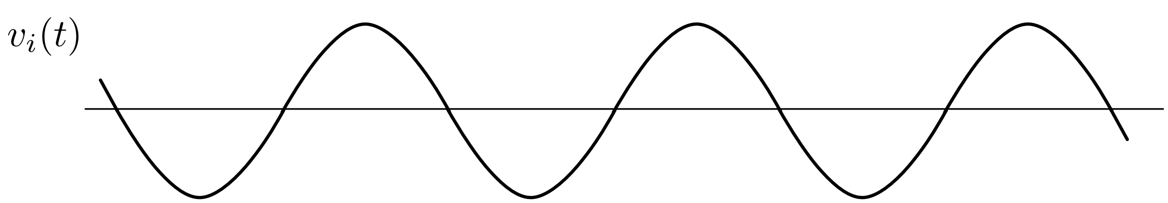
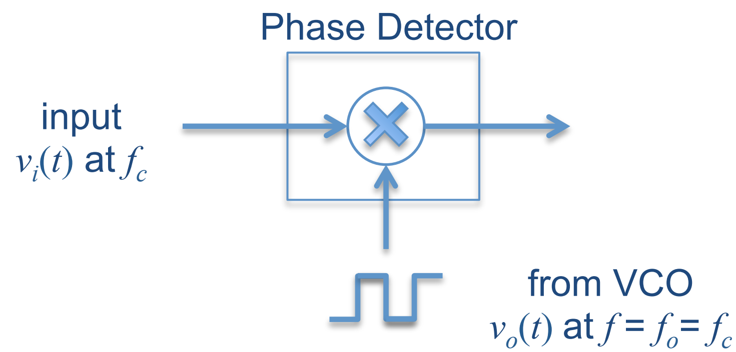


Why the 90^o phase shift ?



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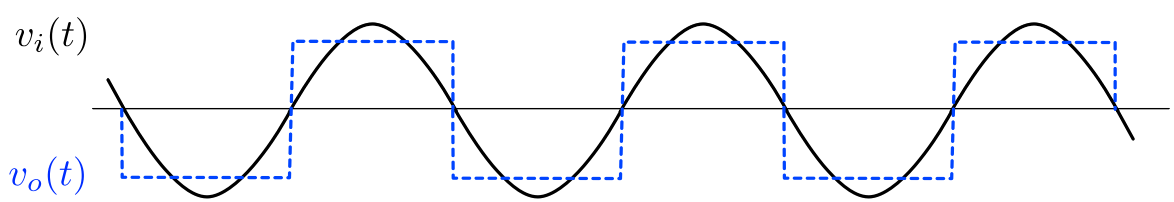

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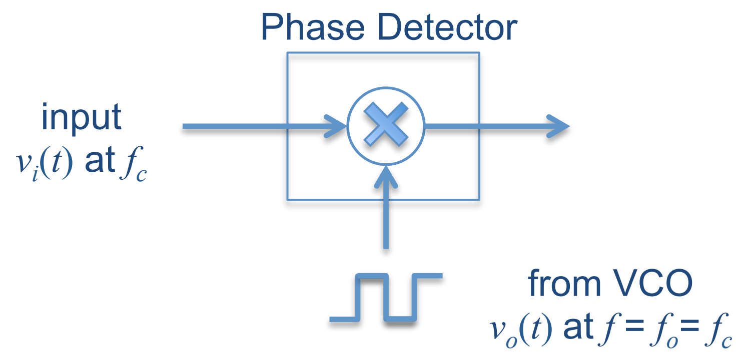

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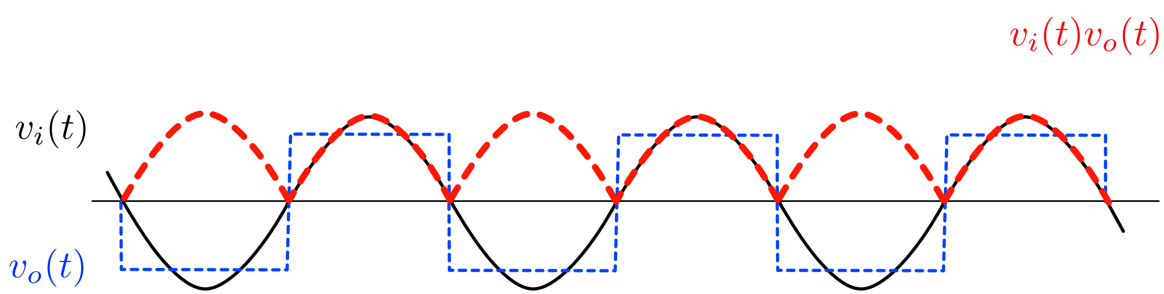

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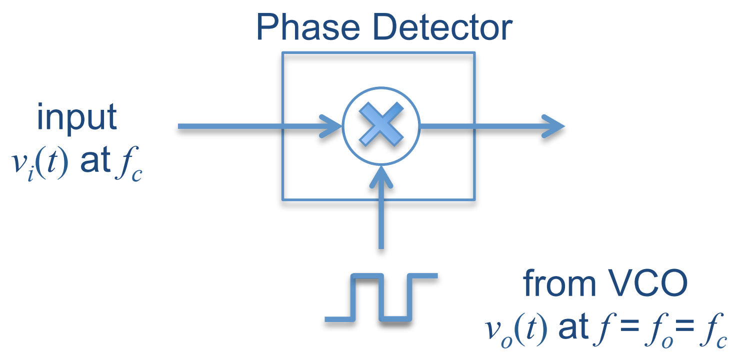

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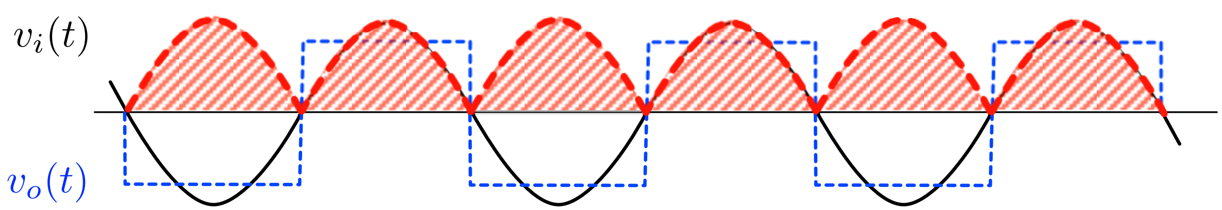
Phase Detector output $\sim$ Area[$v_i(t)v_o(t)$] = maximum



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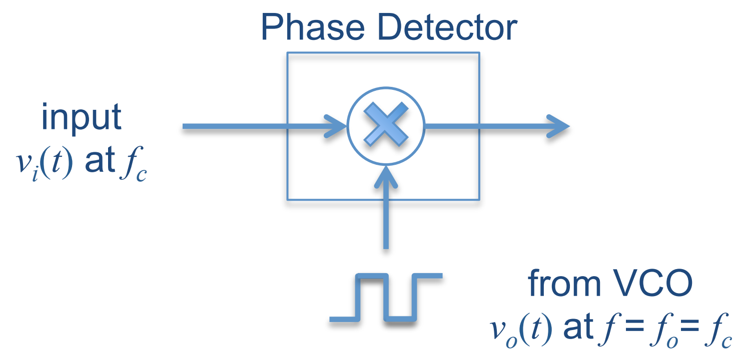
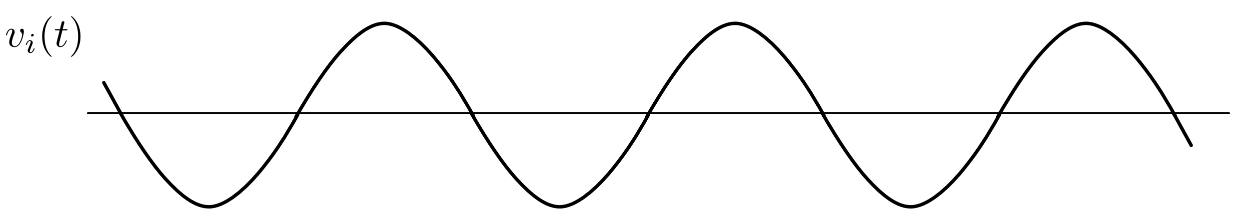


Case 2

$$\left.\begin{aligned} v_i(t) &= \cos 2\pi f_c t \\ v_o(t) &= \sin 2\pi f_o t = \cos(2\pi f_c t - \pi/2) \end{aligned}\right\} \quad \psi_e = \psi_0(t) - \psi_i(t) = -\frac{\pi}{2}$$



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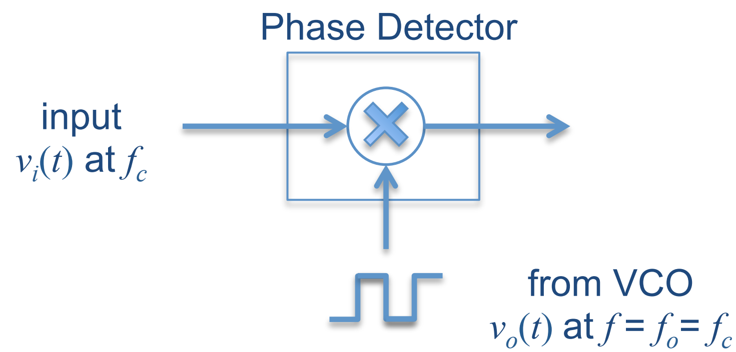

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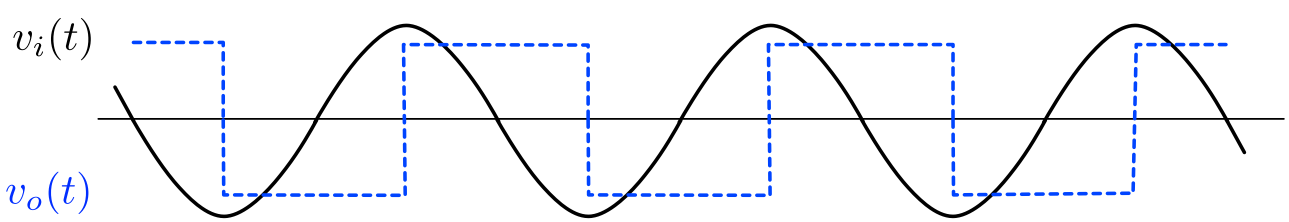

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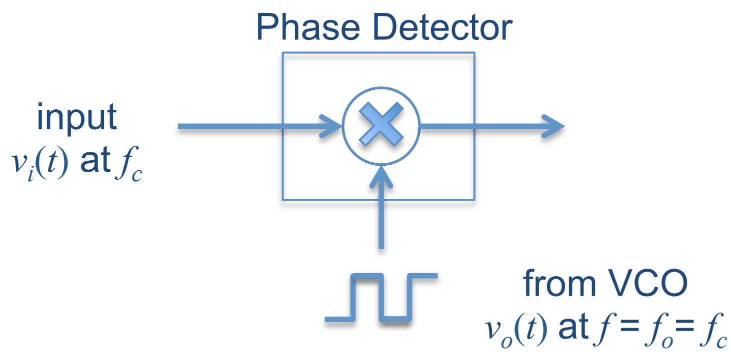

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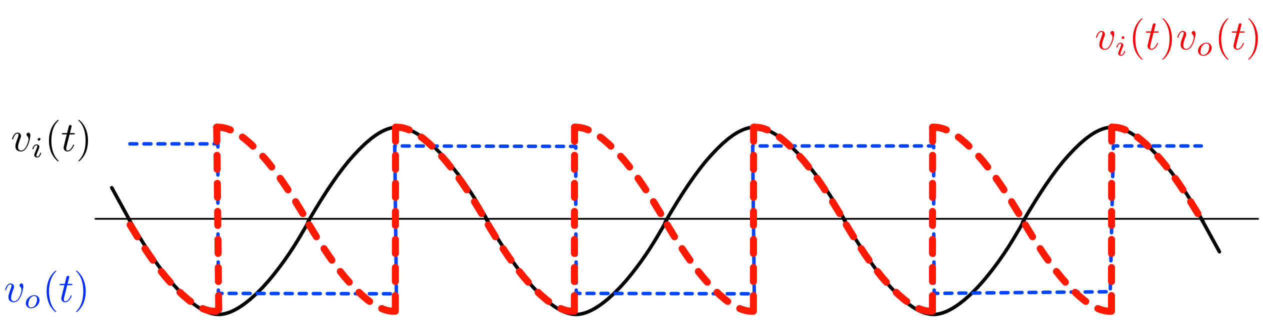

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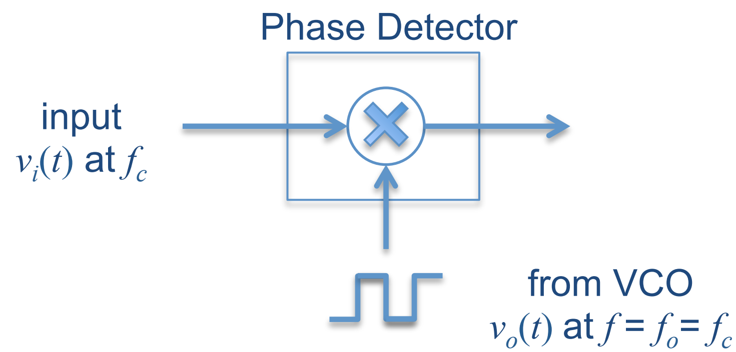

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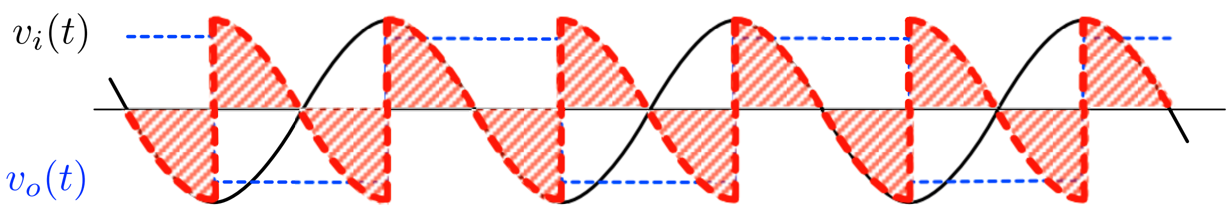
$$\psi_e = \psi_0(t) - \psi_i(t) = -\frac{\pi}{2}$$

Phase Detector output $\sim$ Area[$v_i(t)v_o(t)$] = 0



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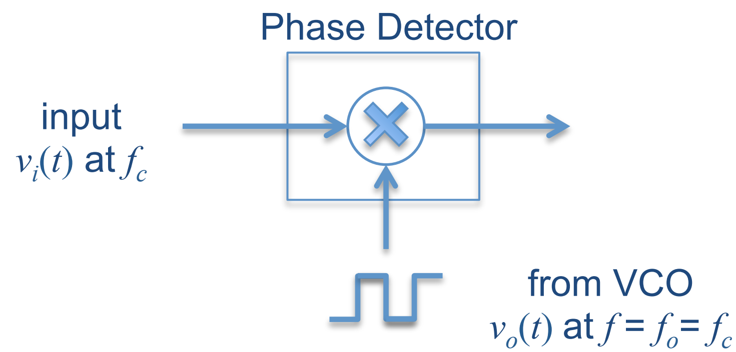
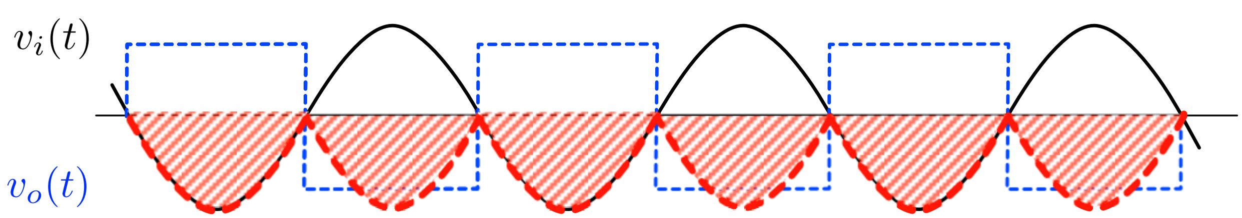


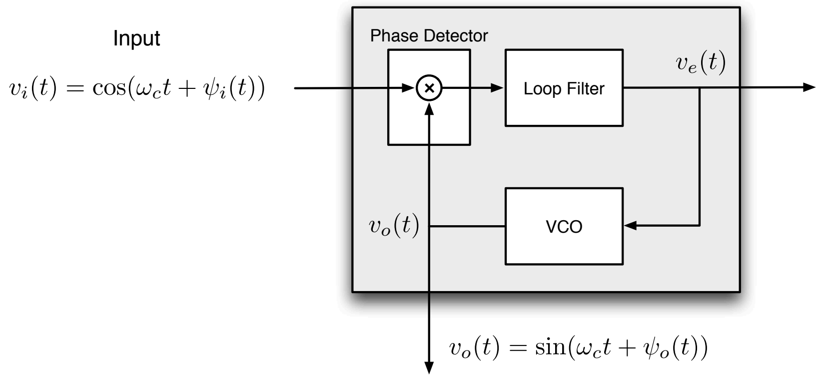
Case 3

$$\left.\begin{aligned} v_i(t) &= \cos 2\pi f_c t \\ v_o(t) &= -\cos 2\pi f_o t = -\cos 2\pi f_c t \end{aligned}\right\} \quad \phi_e = -\pi$$

Phase Detector output $\sim$ Area[$v_i(t)v_o(t)$] = minimum



PLL: Operational parameters



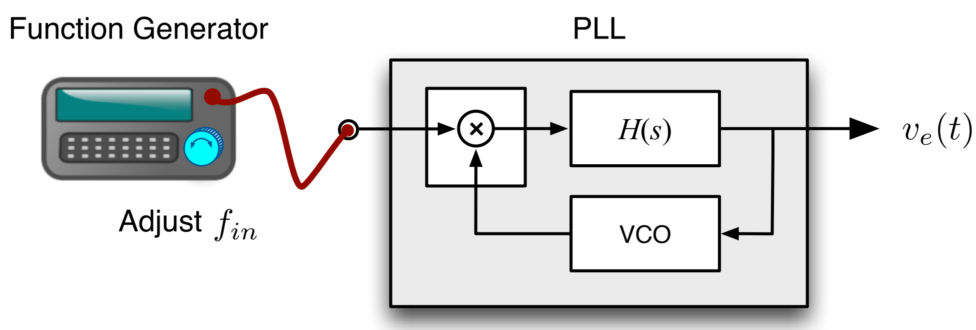
- f_0 **Free-running / Centre frequency**
  frequency at which the VCO operates when not locked to the input signal, i.e., when $v_e(t) = 0$. Pre-set externally.

- f_L **Lock / Tracking / Hold-in range**
  frequency range in the vicinity of f_0 over which the PLL once locked to the input will remain in lock.

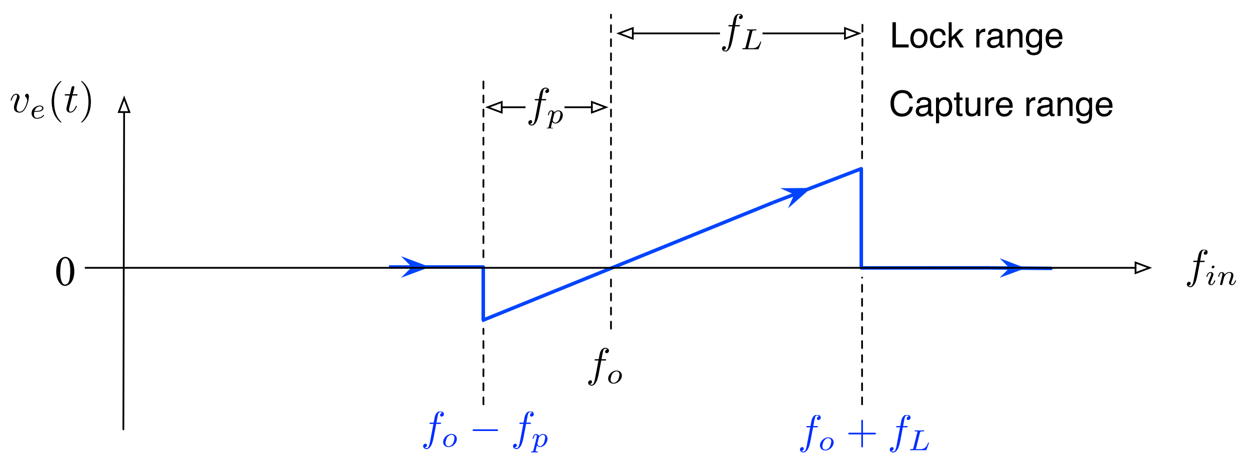
- f_p **Capture / Pull-in / Acquisition range**
  Maximum initial frequency difference between f_0 and f_c $|f_0 - f_c|$ for which the PLL can acquire lock.

PLL: Operational parameters

How to measure ?



- Fix VCO free-running frequency at $f_0 = f_c$
- Change (increase) input frequency f_{in}
- Measure $v_e(t)$



PLL: Operational parameters

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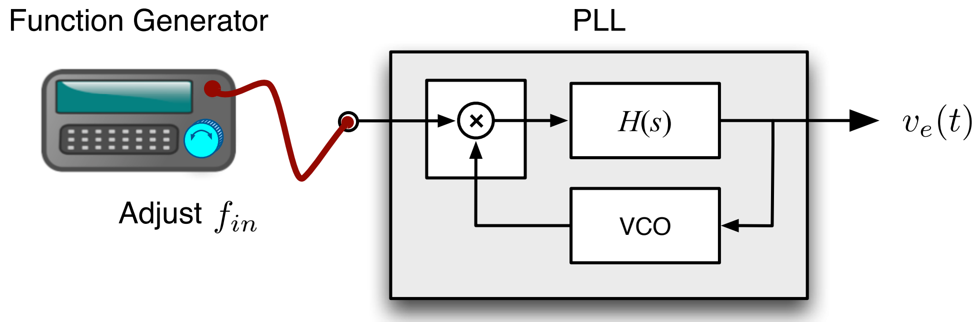
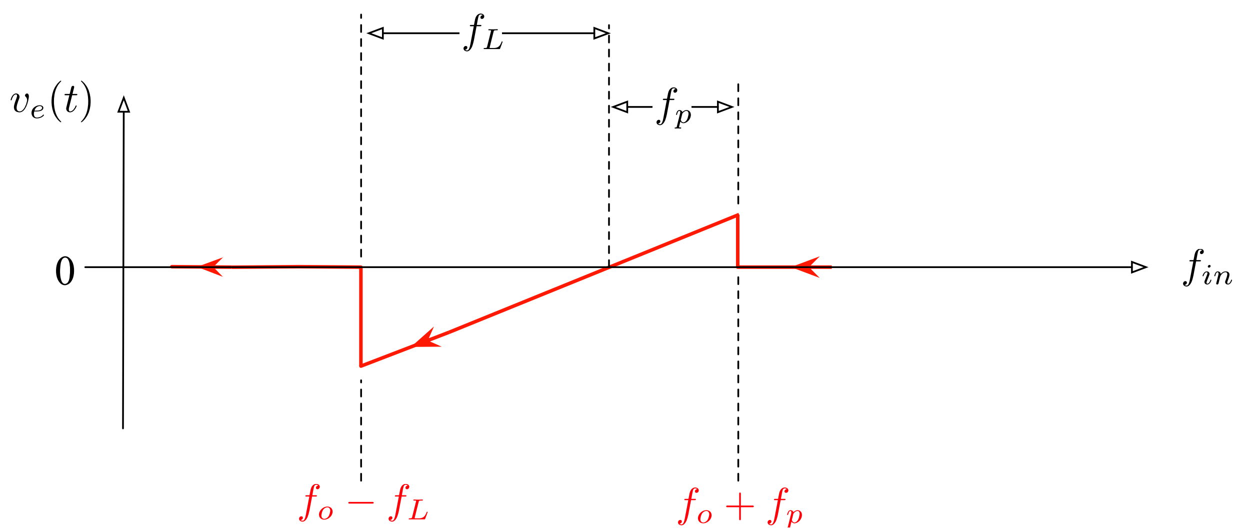


- Fix VCO free-running frequency at $f_0 = f_c$
- Change (decrease) input frequency f_{in}
- Measure $v_e(t)$



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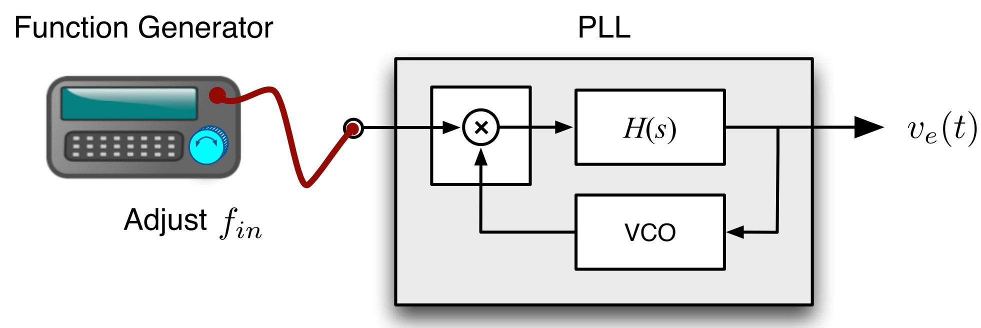
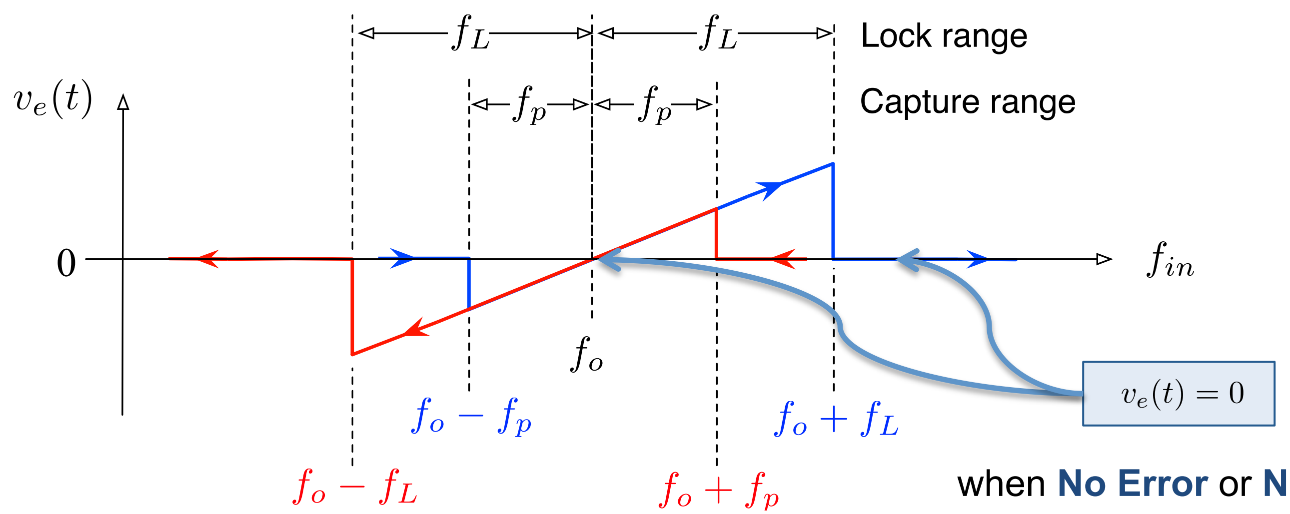


- Fix VCO free-running frequency at $f_0 = f_c$
- Change (increase and decrease) input frequency f_{in}
- Measure $v_e(t)$

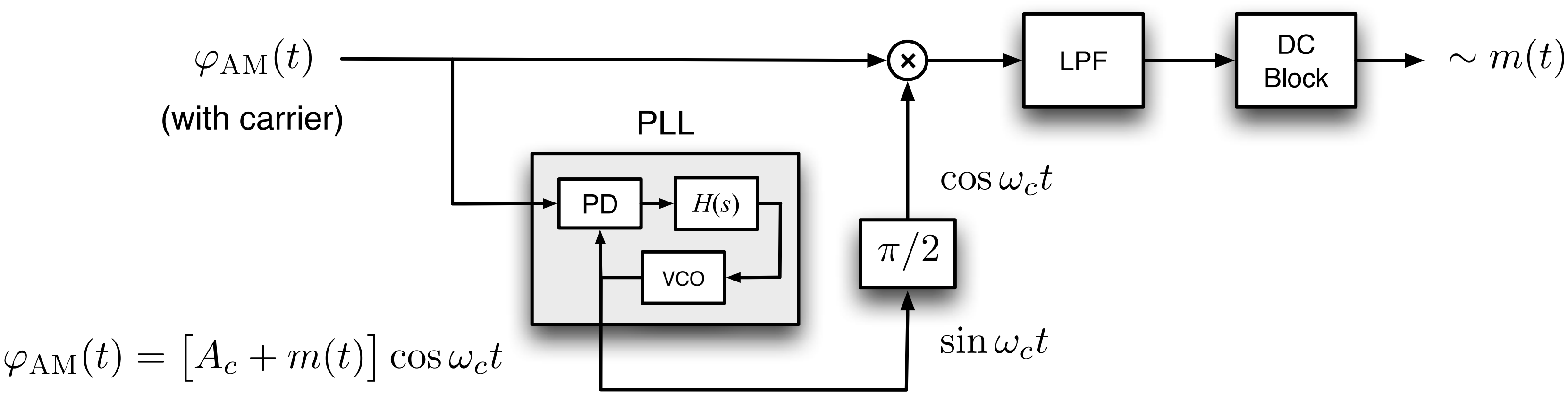


when **No Error** or **No Lock**

Narrow bandwidth PLL: narrow-band tracking / bandpass filter

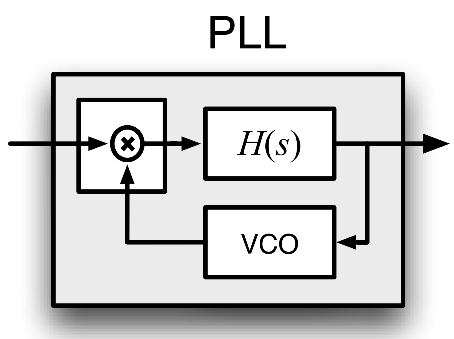
Coherent Demodulation: a modulated waveform with a separate carrier

$$\varphi_{\mathrm{AM}}(t) = \left[\, A_c + m(t) \,\right] \cos \omega_c t$$



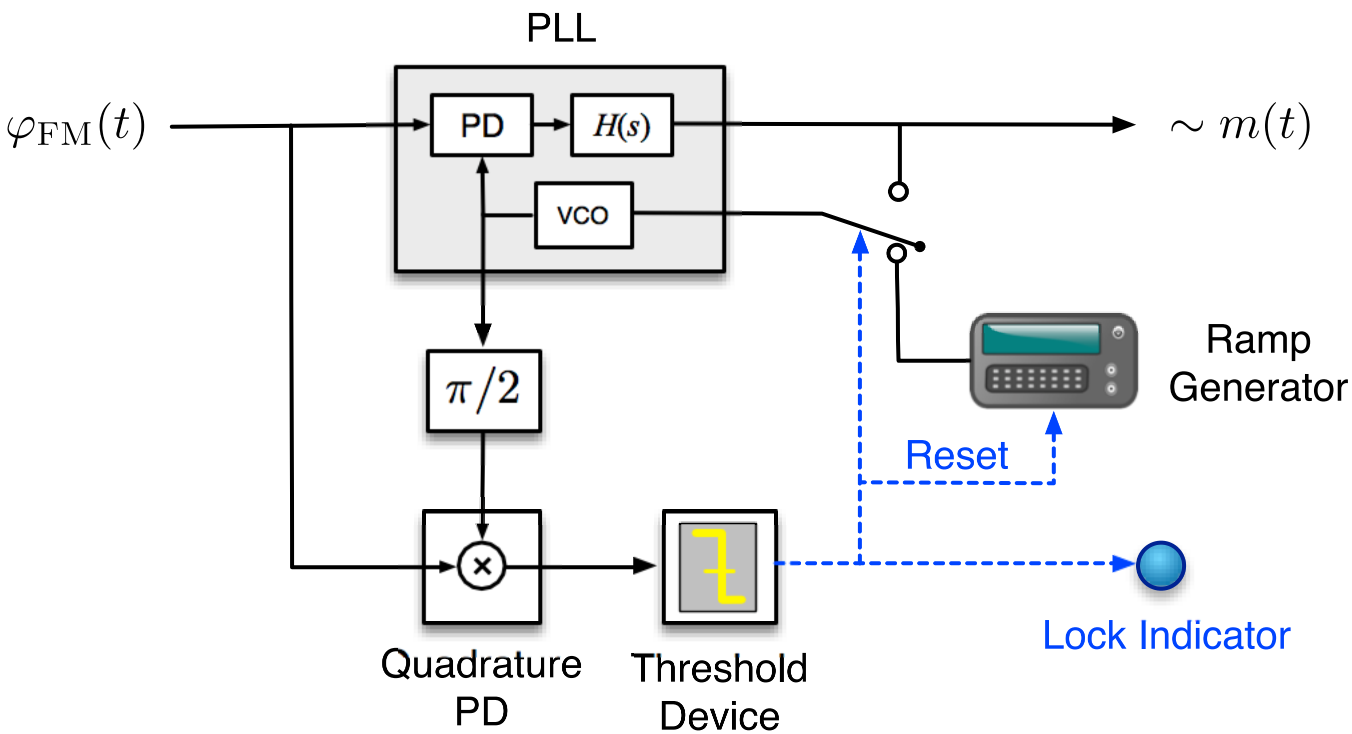
Wide bandwidth PLL: FM demodulation

$$\varphi_{\text{FM}}(t) = \cos\left(\omega_c t + K_f \int m(\lambda) d\lambda\right)$$

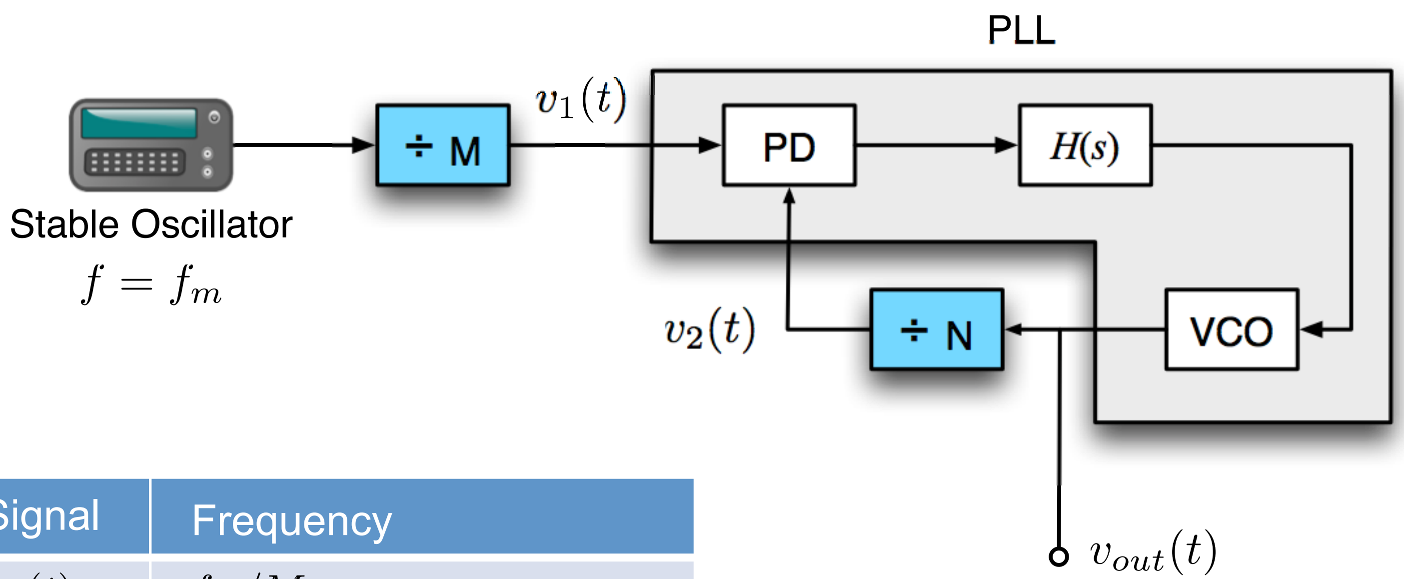


$$\sim m(t)$$

Sweep Acquisition

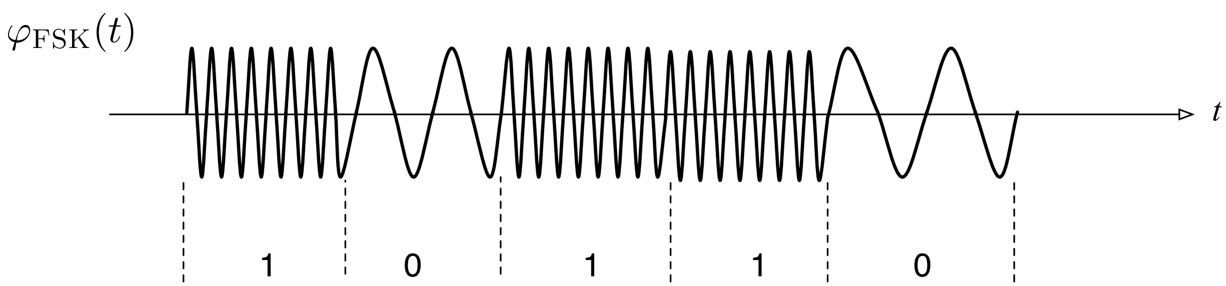


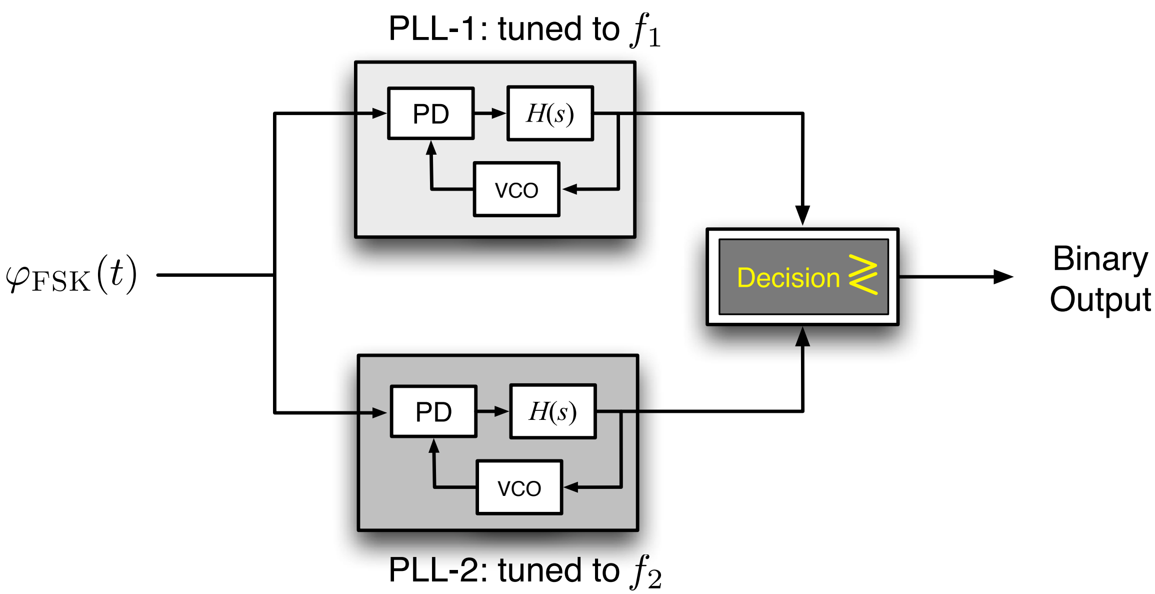
Frequency Synthesis



Signal	Frequency
$v_1(t)$	f_m/M
$v_2(t)$	$f_2 = f_{out}/N = f_m/M$
$v_{out}(t)$	$f_{out} = (N/M)f_m$

Coherent FSK Demodulation





Carrier Acquisition

How to demodulate

- Modulated signals with a carrier (DSB-LC, SSB+C, VSB+C …)

non-coherent / envelope detection or **coherent detection**

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  coherent detection

How to demodulate

- Modulated signals with a carrier (DSB-LC, SSB+C, VSB+C …)

  non-coherent / envelope detection or **coherent detection**

- Suppressed carrier modulated signals (DSB-SC, SSB, VSB …)

  coherent detection

- For coherent (or synchronous) detection we need to **generate a local carrier** at the receiver.
- Any discrepancy in the frequency or phase of the local carrier creates distortion in the detector output.

Carrier Acquisition is the process of **generating a local carrier** in the receiver that is **frequency and phase synchronized** with the carrier used in the transmitter.

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An alternative:

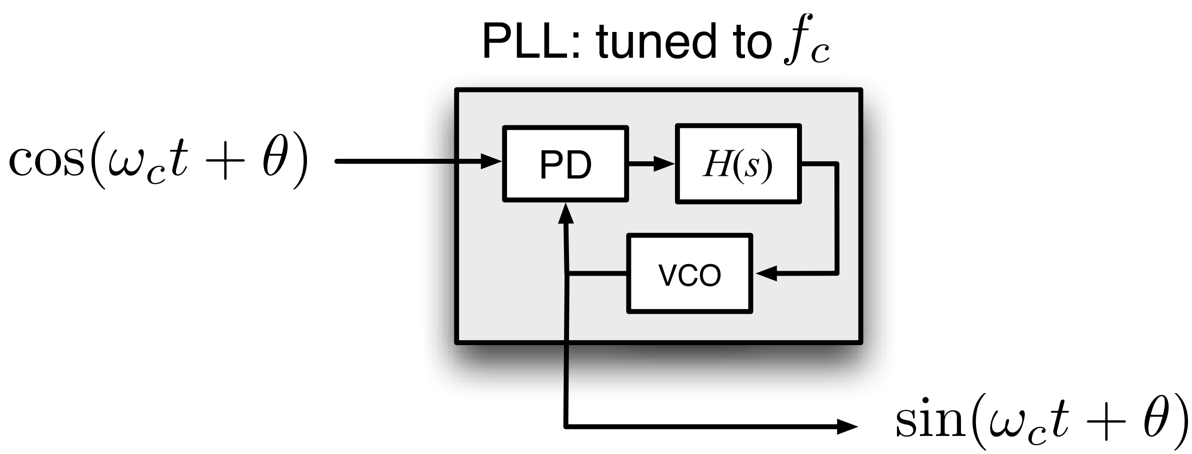
- Add a separate carrier/pilot to the modulated signal at a reduced level (typically 20 dB below the signal power).
- Converts suppressed carrier signal into a small carrier format.
- This may be the only viable solution to synchronize RX with TX.

Of course, at the RX we still need the signal processing components to lock onto and track the pilot tone.

Carrier Acquisition for Signals with a Carrier

PLL can be used to track both the frequency and the phase of the carrier.

- Used for **coherent**/synchronous **demodulation of DSB-LC/AM signals**
- Can also used in wide-band PLL mode for **demodulating FM/PM signals**.



Carrier Acquisition from DSB-SC Signals: Signal Squaring

A DSB-SC amplitude modulated waveform of the form

$$\varphi_{\text{DSB-SC}} = m(t)\cos\omega_c t$$

does not have a separate carrier term.

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$$\varphi_{\text{DSB-SC}} = m(t)\cos\omega_c t$$

does not have a separate carrier term.

But … with the help of trigonometric identities and some clever signal processing we may still generate a phase coherent carrier from the received waveform at the receiver.

Carrier Acquisition from DSB-SC Signals: Signal Squaring

Let

$$m(t): \text{ zero-mean signal}$$

$$m^2(t): \text{ non zero-mean signal}$$

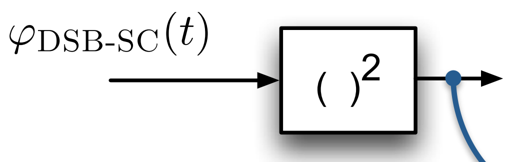
and assume

$$m^2(t) = K + x_\phi(t)$$

K: d.c. level of $m^2(t)$

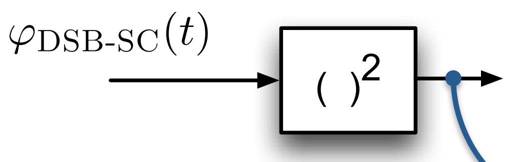
$x_\phi(t)$: zero-mean component of $m^2(t)$

Carrier Acquisition from DSB-SC Signals: Signal Squaring



$$\varphi_{\text{DSB-SC}}(t) = m^2(t)\cos^2\omega_c t$$

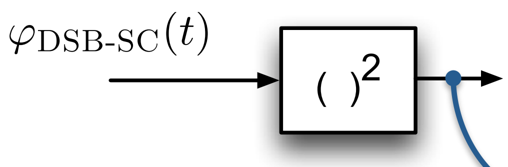
Carrier Acquisition from DSB-SC Signals: Signal Squaring



$$\varphi_{\text{DSB-SC}}(t) = m^2(t)\cos^2\omega_c t$$

$$= \frac{1}{2}m^2(t) + \frac{1}{2}m^2(t)\cos 2\omega_c t$$

Carrier Acquisition from DSB-SC Signals: Signal Squaring



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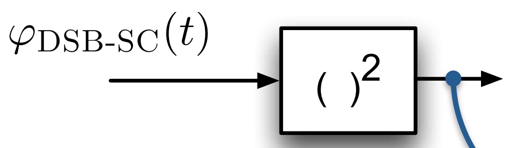
$$= \frac{1}{2}m^2(t) + \frac{1}{2}\left[K + x_\phi(t)\right]\cos 2\omega_c t$$

Carrier Acquisition from DSB-SC Signals: Signal Squaring



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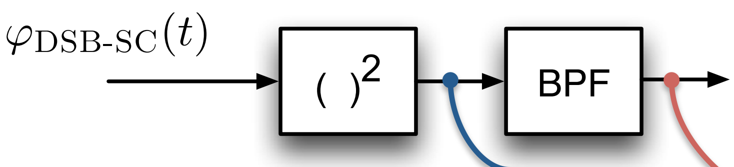
$$= \frac{1}{2}m^2(t) + \frac{1}{2}K\cos 2\omega_c t + \frac{1}{2}x_\phi(t)\cos 2\omega_c t$$

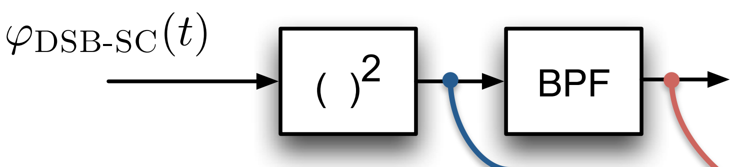
Carrier Acquisition from DSB-SC Signals: Signal Squaring



$$\varphi^2_{\text{DSB-SC}}(t) = \frac{1}{2}m^2(t) + \frac{1}{2}K\cos 2\omega_c t + \frac{1}{2}x_\phi(t)\cos 2\omega_c t$$

$$\varphi^2_{\text{DSB-SC}}(t) * h_{\text{BPF}}(t) = \left[\frac{1}{2}m^2(t) + \frac{1}{2}K\cos 2\omega_c t + \frac{1}{2}x_\phi(t)\cos 2\omega_c t\right] * h_{\text{BPF}}(t)$$

Carrier Acquisition from DSB-SC Signals: Signal Squaring



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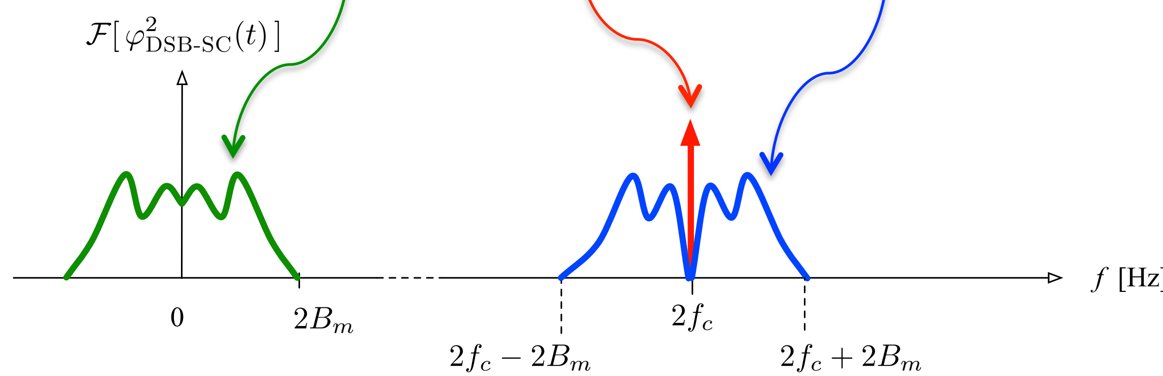

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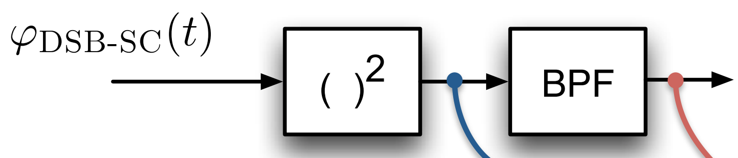

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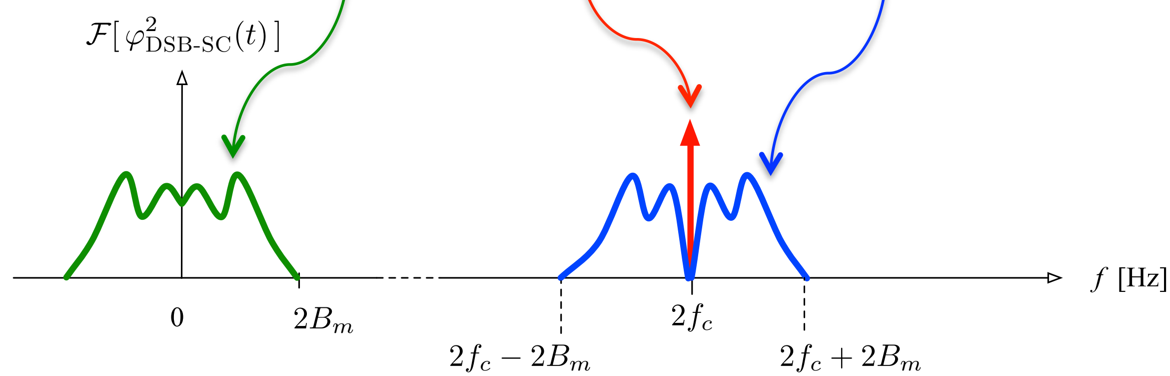
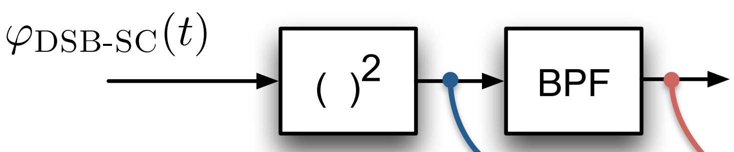


Carrier Acquisition from DSB-SC Signals: Signal Squaring



$$\varphi^2_{\text{DSB-SC}}(t) = \frac{1}{2}m^2(t) + \frac{1}{2}K\cos 2\omega_c t + \frac{1}{2}x_\phi(t)\cos 2\omega_c t$$

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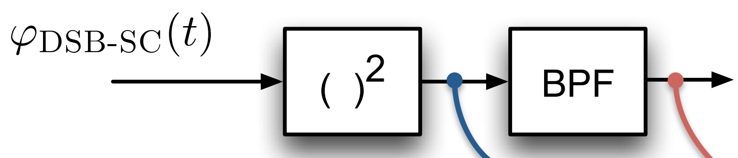

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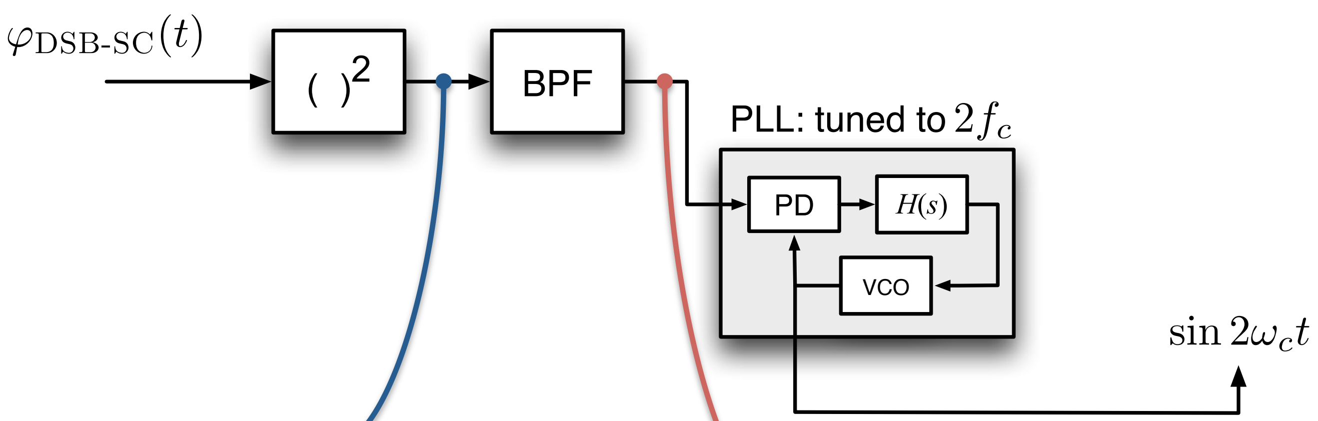
$$= \frac{1}{2}K\cos 2\omega_c t + [\text{ small residual }]$$

$$\approx \frac{1}{2}K\cos 2\omega_c t$$

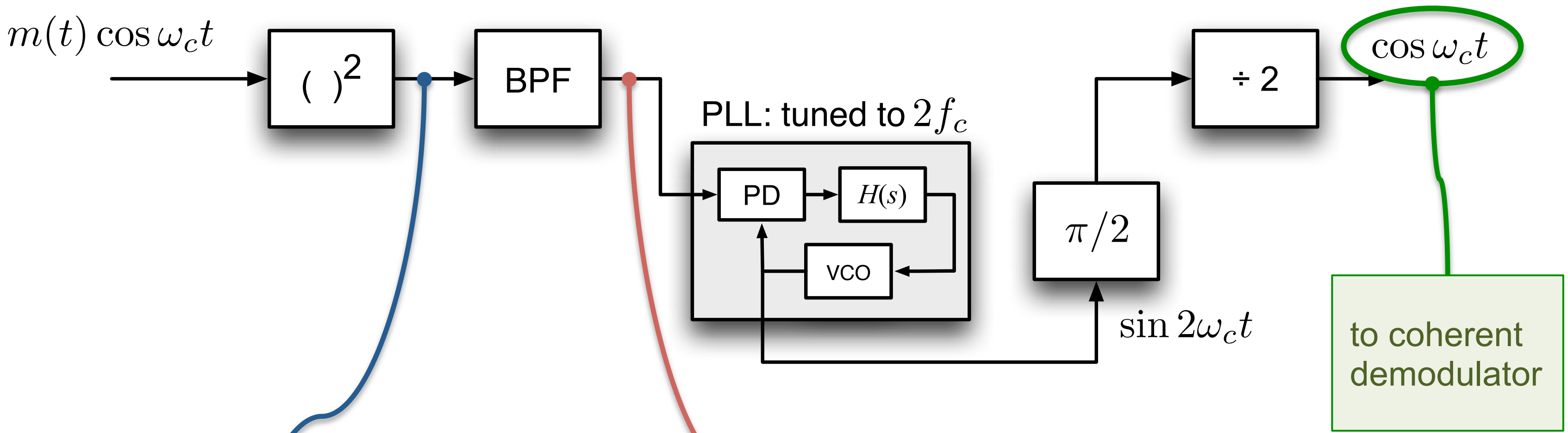
Carrier Acquisition from DSB-SC Signals: Signal Squaring



$$\varphi^2_{\text{DSB-SC}}(t) * h_{\text{BPF}}(t) \approx \frac{1}{2} K \cos 2\omega_c t$$

$$\varphi^2_{\text{DSB-SC}}(t) = \frac{1}{2} m^2(t) + \frac{1}{2} K \cos 2\omega_c t + \frac{1}{2} x_\phi(t) \cos 2\omega_c t$$

Carrier Acquisition from DSB-SC Signals: Signal Squaring



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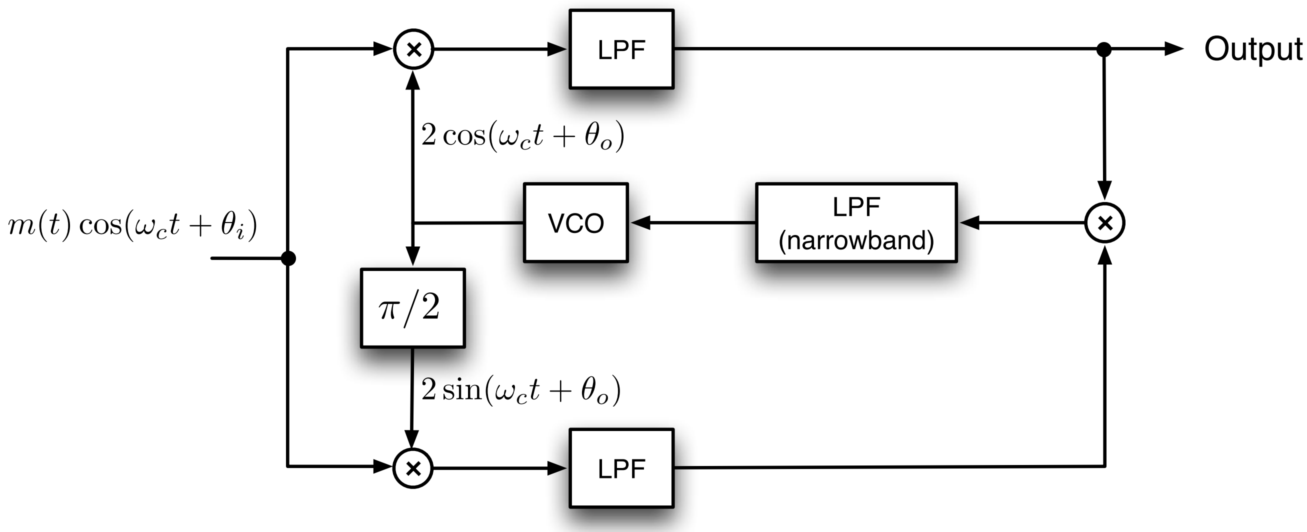
Carrier Acquisition from DSB-SC Signals: Costas Loop

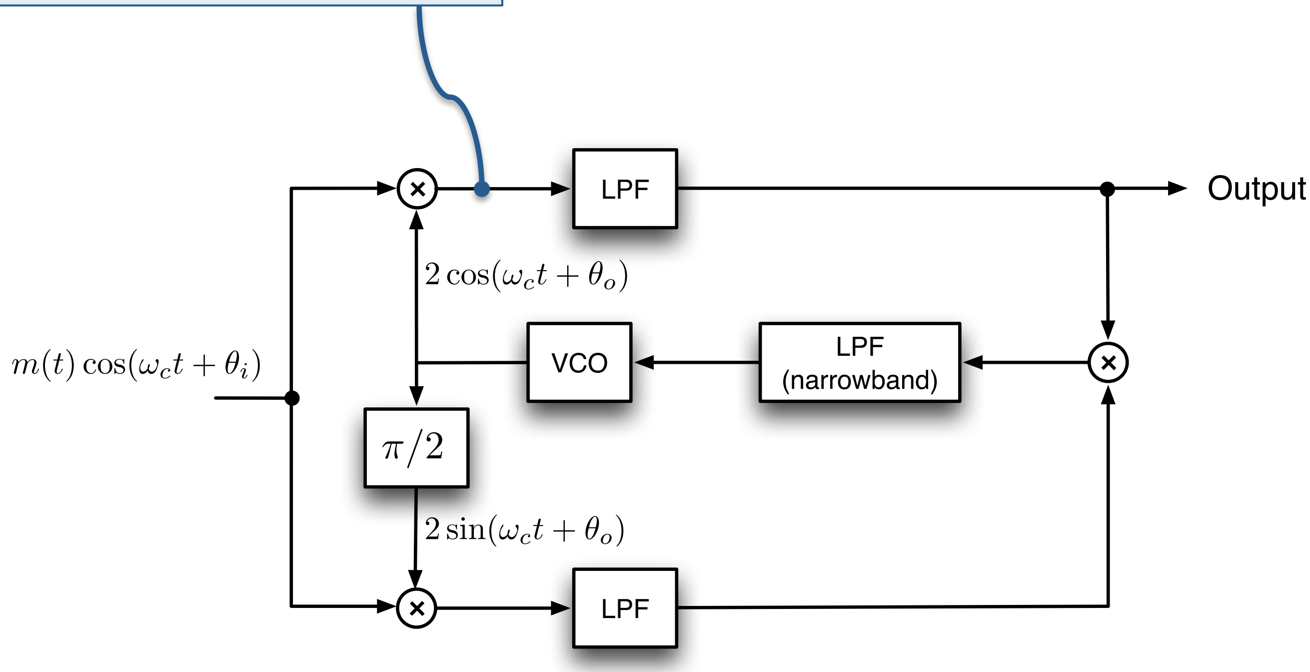


Initialization

- Input: $\varphi_{\text{DSB-SC}}(t) = m(t)\cos(\omega_c t + \theta_i)$
- VCO adjusted to generate a sinusoid at the carrier frequency f_c and with an arbitrary/random phase θ_o: $\cos(\omega_c t + \theta_o)$

Carrier Acquisition from DSB-SC Signals: Costas Loop

$$2m(t)\cos(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o) = m(t)\cos(\theta_i - \theta_o) + m(t)\cos(2\omega_c t + \theta_i + \theta_o)$$



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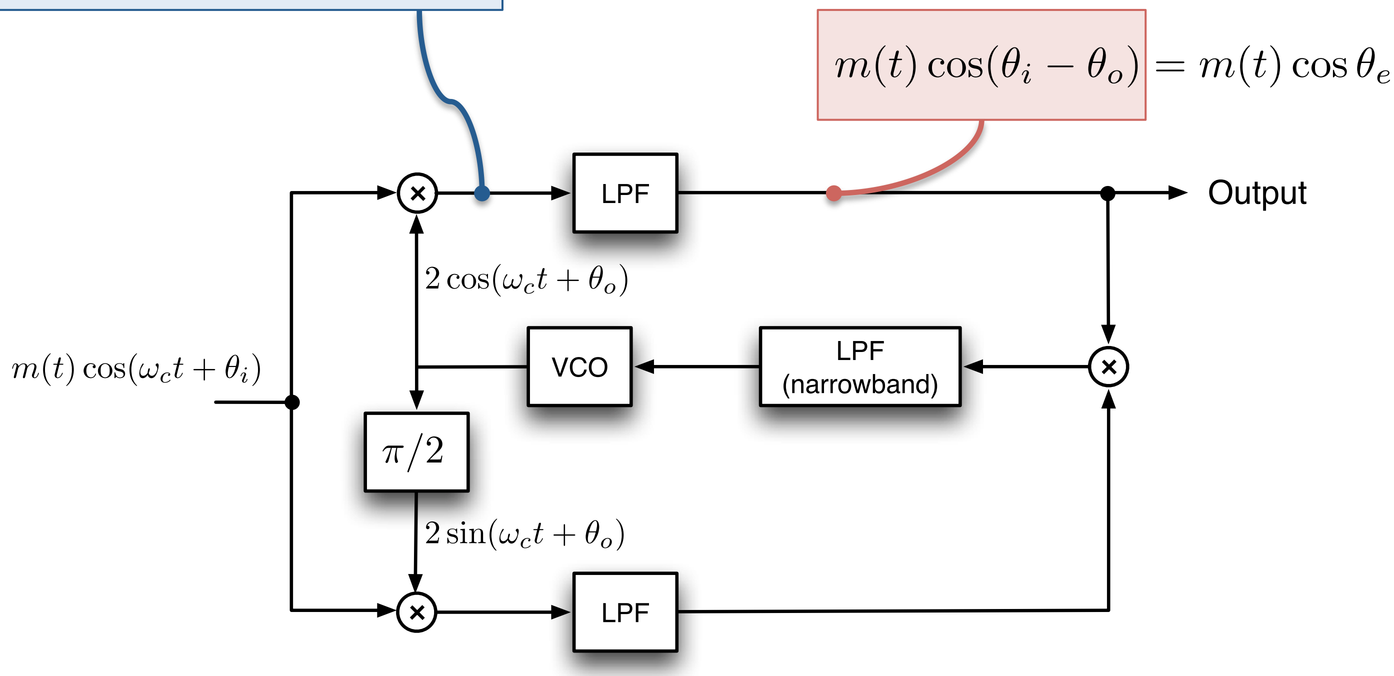
$$m(t)\cos(\theta_i - \theta_o) = m(t)\cos\theta_e$$



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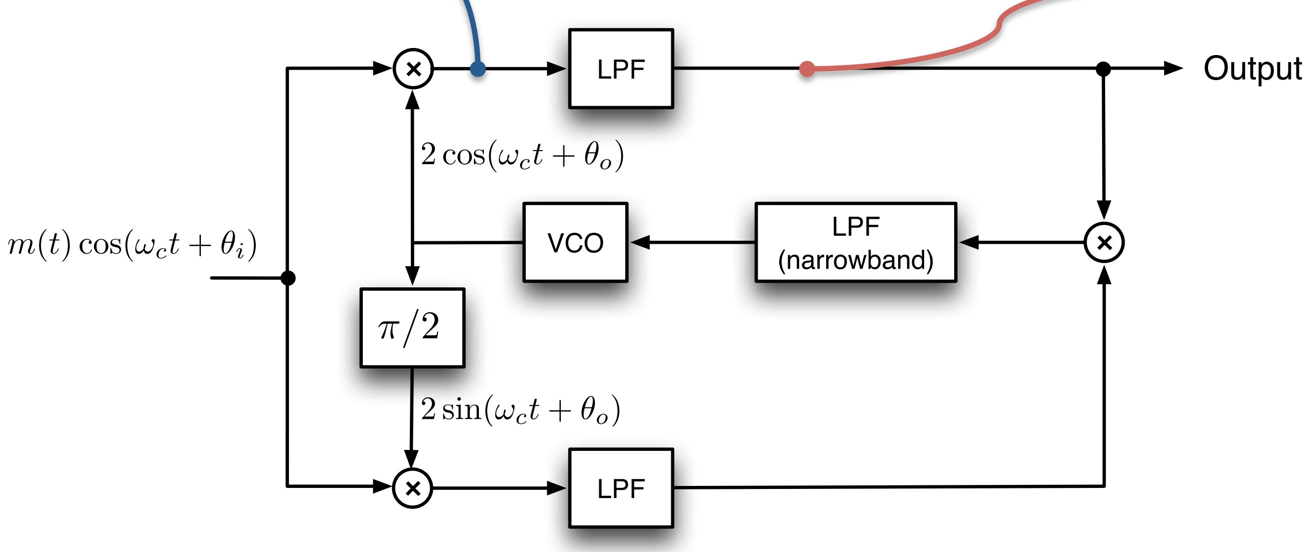
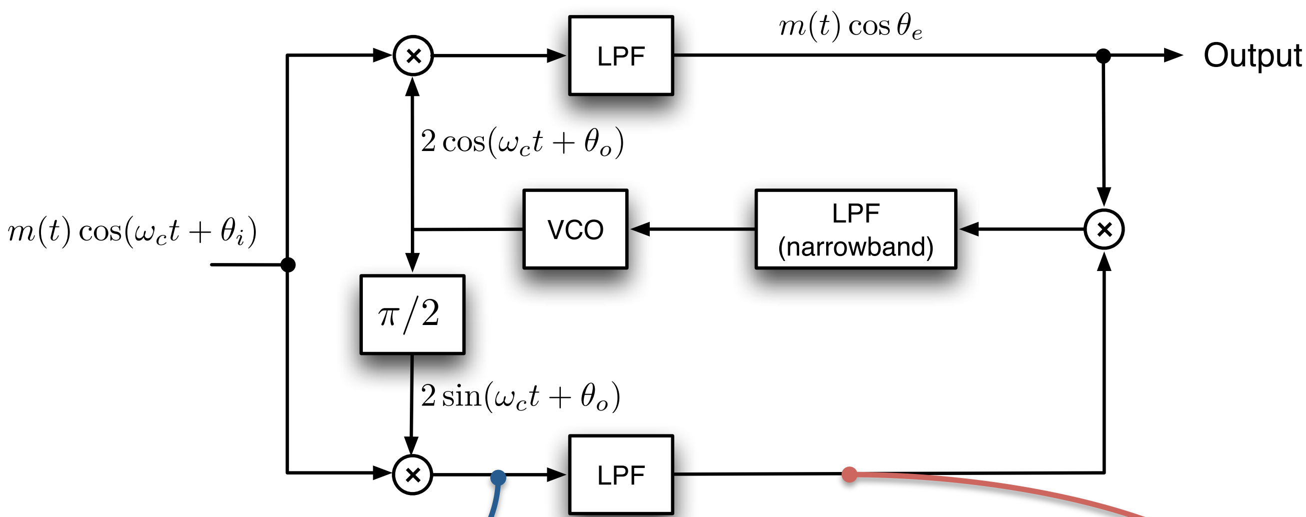


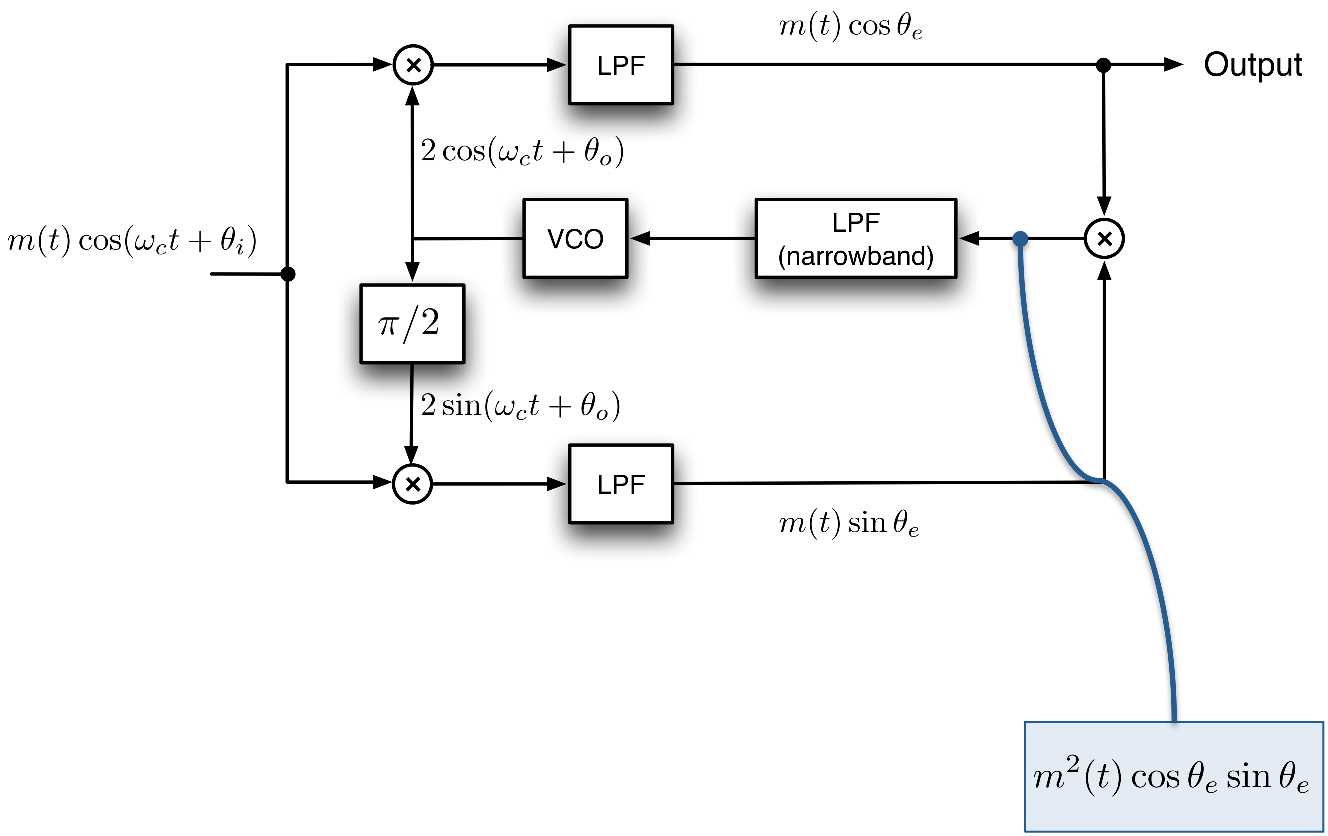
Carrier Acquisition from DSB-SC Signals: Costas Loop



$$m(t)\sin(\theta_i - \theta_o) = m(t)\sin\theta_e$$

$$2m(t)\cos(\omega_c t + \theta_i)\sin(\omega_c t + \theta_o) = m(t)\sin(\theta_i - \theta_o) + \cancel{m(t)\sin(2\omega_c t + \theta_i + \theta_o)}$$

Carrier Acquisition from DSB-SC Signals: Costas Loop



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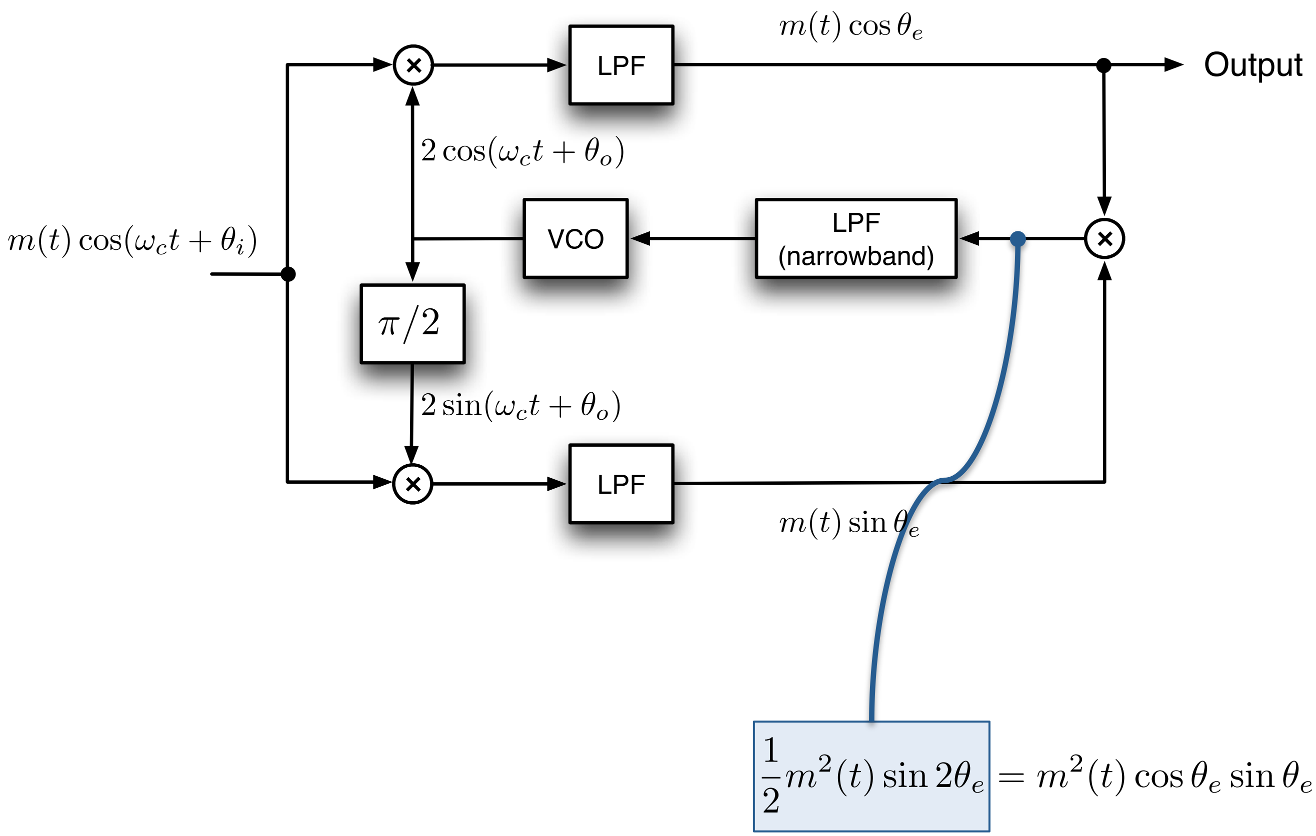

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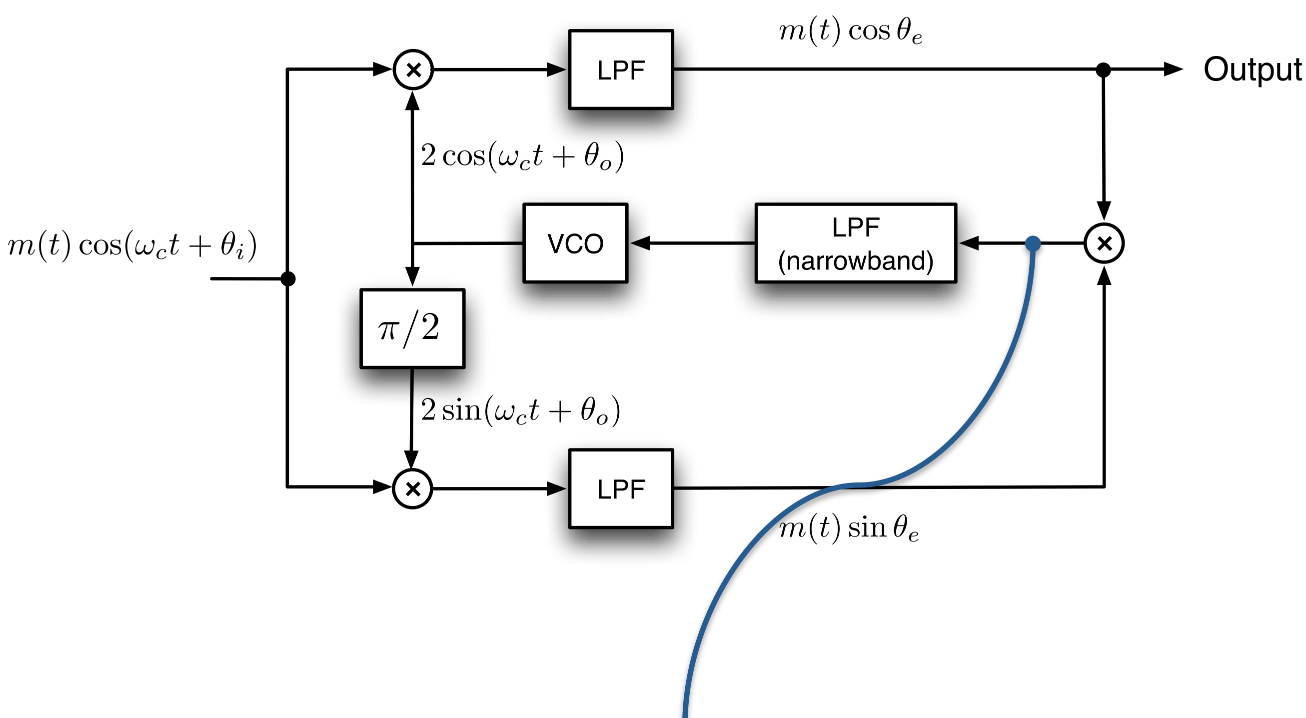
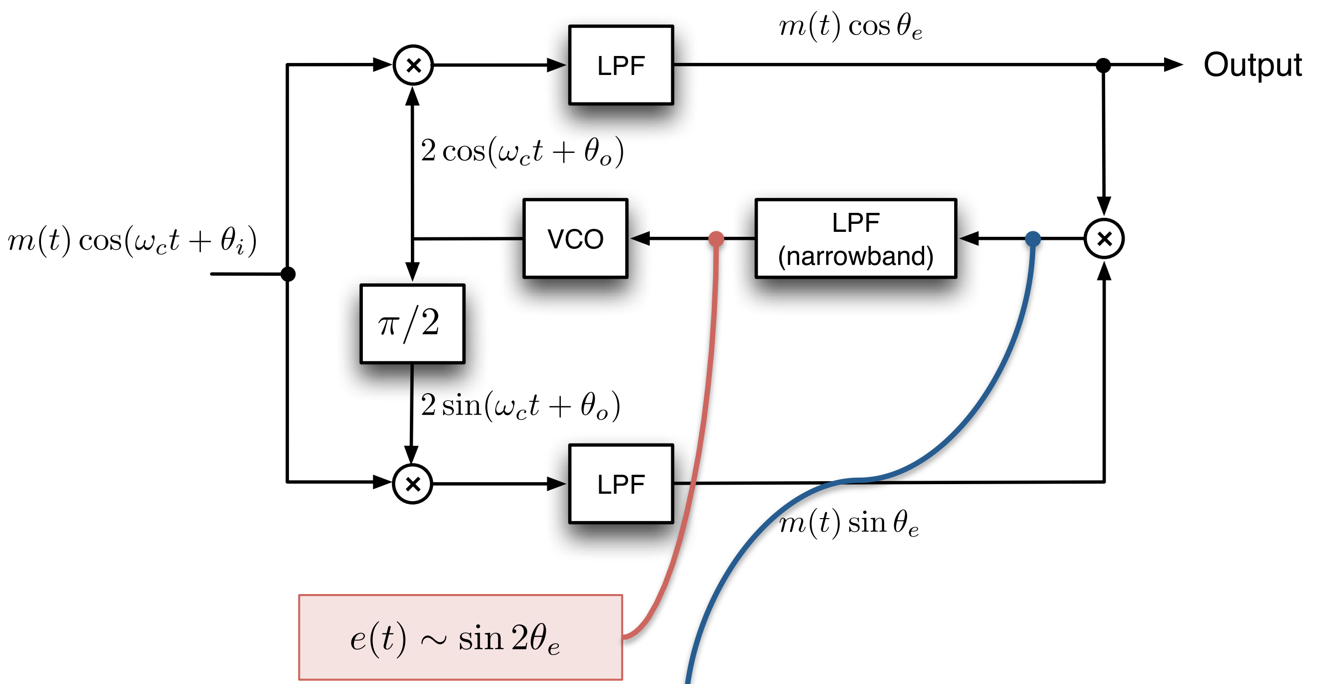


$$K\sin 2\theta_e + \frac{1}{2}x_\phi(t)\sin 2\theta_e = \frac{1}{2}m^2(t)\sin 2\theta_e = m^2(t)\cos\theta_e\sin\theta_e$$

Carrier Acquisition from DSB-SC Signals: Costas Loop



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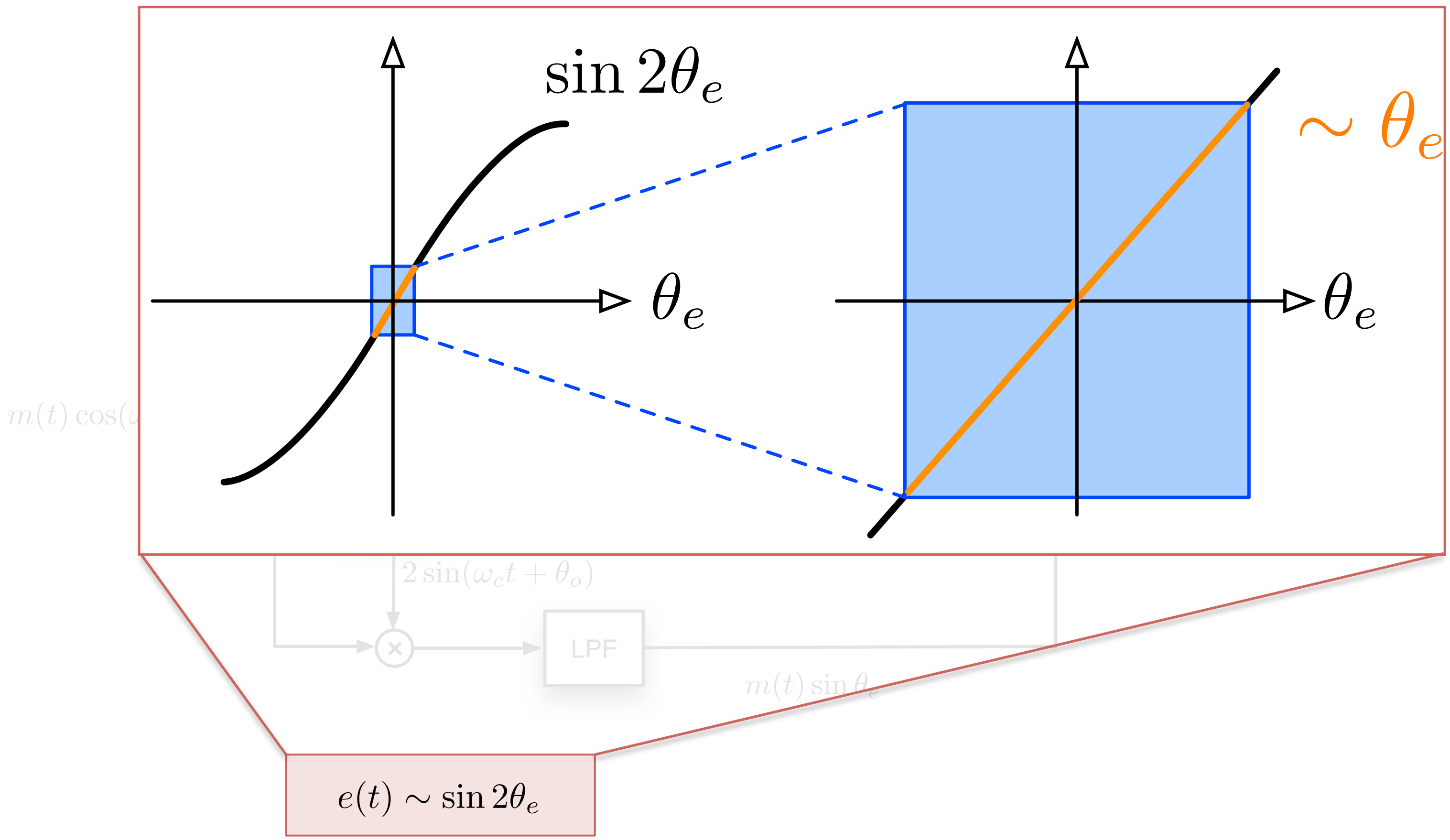

$$K \sin 2\theta_e + \frac{1}{2} x_\phi(t) \sin 2\theta_e = \frac{1}{2} m^2(t) \sin 2\theta_e = m^2(t) \cos\theta_e \sin\theta_e$$

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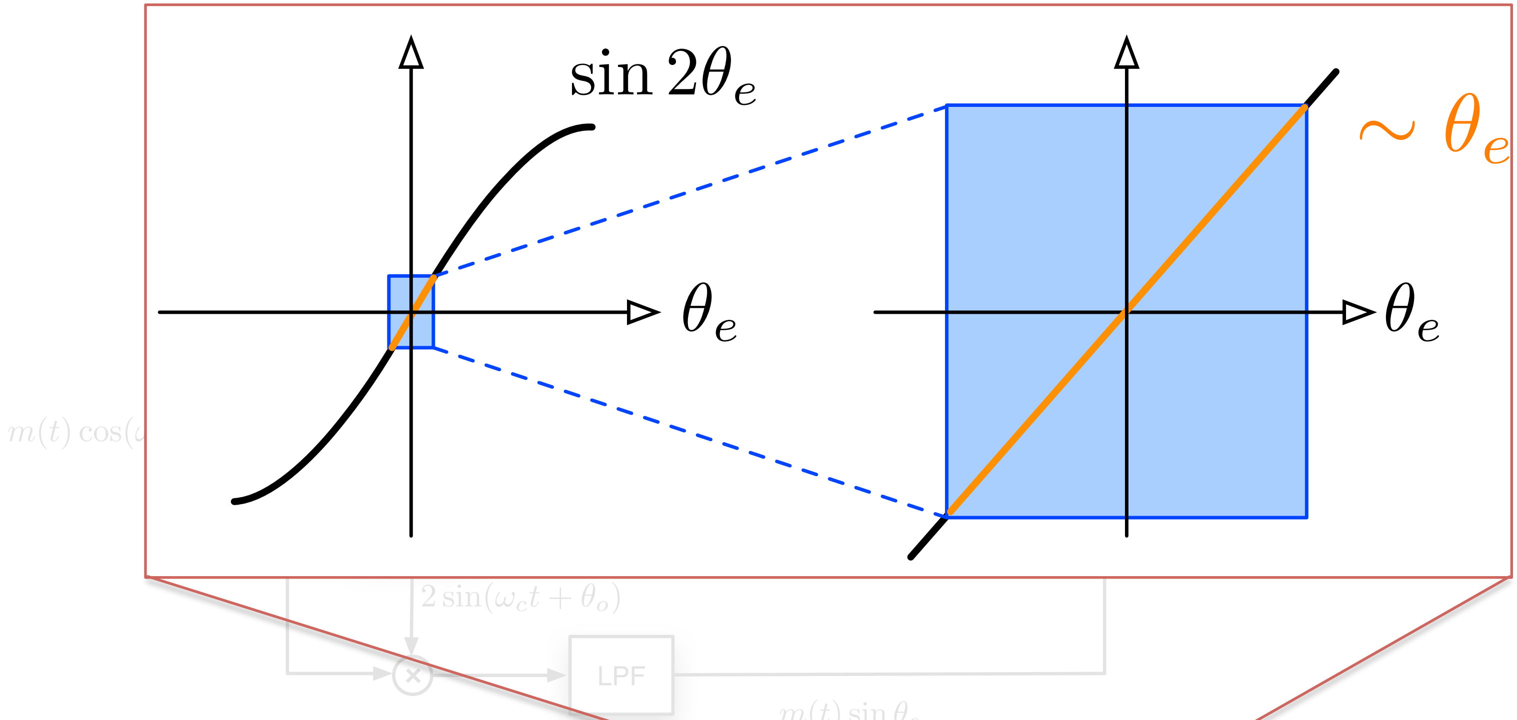
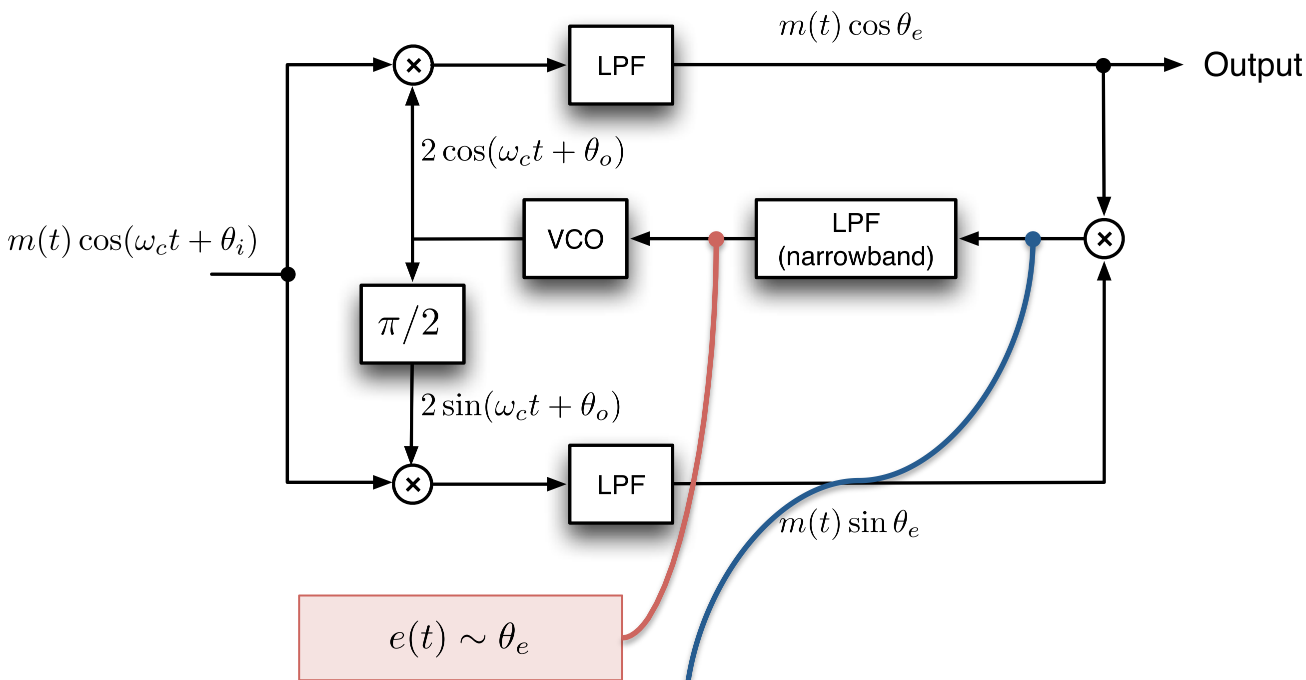


$e(t) \sim \sin 2\theta_e$ → $e(t) \sim \theta_e$

$$K \sin 2\theta_e + \frac{1}{2} x_\phi(t) \sin 2\theta_e = \frac{1}{2} m^2(t) \sin 2\theta_e = m^2(t) \cos\theta_e \sin\theta_e$$

Carrier Acquisition from DSB-SC Signals: Costas Loop



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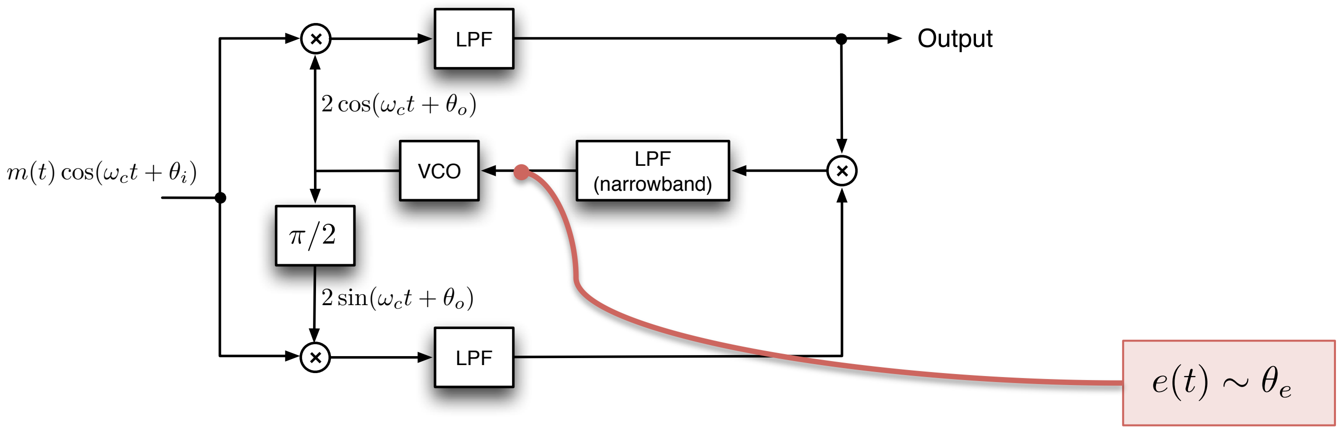

Based on the preceding analysis, we generated a feedback system with:

- VCO input : $e(t) \sim \theta_e = \theta_i - \theta_o$
- VCO output: changes to drive the error signal to zero

$e(t) \longrightarrow 0$ → $\theta_o \longrightarrow \theta_i$ → phase and frequency coherent carrier